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CHAPTER 4

Methods of Analysis of Resistive Circuits

IN THIS CHAPTER:

4.1 Introduction

To analyze an electric circuit, we write and solve a set of equations. We apply Kirchhoff's current and voltage laws to get some of the equations. The constitutive equations of the circuit elements, such as Ohm's law, provide the remaining equations. The unknown variables are element currents and voltages. Solving the equations provides the values of the element current and voltages.

This method works well for small circuits, but the set of equations can get quite large for even moderate-sized circuits. A circuit with only 6 elements has 6 element currents and 6 element voltages. We could have 12 equations in 12 unknowns. In this chapter we consider two methods for writing a smaller set of simultaneous equations:

- The node voltage method
- The mesh current method

The node voltage method uses a new type of variable called the node voltage. The "node voltage equations" or, more simply, the "node equations" are a set of simultaneous equations that represent a given electric circuit. The unknown variables of the node voltage equations are the node voltages. After solving the node voltage equations, we determine the values of the element currents and voltages from the values of the node voltages.

It's easier to write node voltage equations for some types of circuit than for others. Starting with the easiest case, we will learn how to write node voltage equations for circuits that consist of:

- Resistors and independent current sources
- Resistors and independent current and voltage sources
- Resistors and independent and dependent voltage and current sources

The mesh current method uses a new type of variable called the mesh current. The "mesh current equations" or, more simply, the "mesh equations" are a set of simultaneous equations that represent a given electric circuit. The unknown variables of the mesh current equations are the mesh currents. After solving the mesh current equations, we determine the values of the element currents and voltages from the values of the mesh currents.

It's easier to write mesh current equations for some types of circuit than for others. Starting with the easiest case, we will learn how to write mesh current equations for circuits that consist of:

- Resistors and independent voltage sources
- Resistors and independent current and voltage sources
- Resistors and independent and dependent voltage and current sources

4.2 Node Voltage Analysis of Circuits with Current Sources

Consider the circuit shown in Figure 4.2-1*a*. This circuit contains four elements: three resistors and a current source. The *nodes* of a circuit are the places where the elements are connected together. The circuit shown in Figure 4.2-1*a* has three nodes. It is customary to draw the elements horizontally or vertically and to connect these elements by horizontal and vertical lines that represent wires. In other words, nodes are drawn as points, or are drawn using horizontal or vertical lines. Figure 4.2-1*b* shows the same circuit, redrawn so that all three nodes are drawn as points rather than lines. In Figure 4.2-1*b* the nodes are labeled as node a, node b, and node c.

Analyzing a connected circuit containing n nodes will require $n - 1$ KCL equations. One way to obtain these equations is to apply KCL at each node of the circuit except for one. The

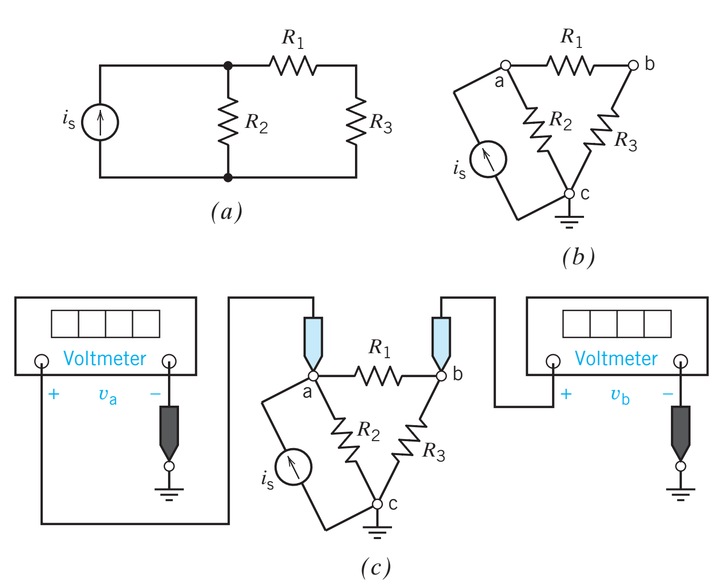

FIGURE 4.2-1 (*a*) A circuit with three nodes. (*b*) The circuit after the nodes have been labeled and a reference node has been selected and marked. (*c*) Using voltmeters to measure the node voltages.

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node at which KCL is not applied is called the reference node. Any node of the circuit can be selected to be the reference node. We will often choose the node at the bottom of the circuit to be the reference node. (When the circuit contains a grounded power supply, the ground node of the power supply is usually selected as the reference node.) In Figure 4.2-1*b*, node c is selected as the reference node and marked with the symbol used to identify the reference node.

The voltage at any node of the circuit, relative to the reference node, is called a **node voltage.** In Figure 4.2-1*b*, there are two node voltages: the voltage at node a with respect to the reference node, node c, and also the voltage at node b, again with respect to the reference node, node c. In Figure 4.2-1*c*, voltmeters are added to measure the node voltages. To measure node voltage at node a, connect the red probe of the voltmeter at node a and connect the black probe at the reference node, node c. To measure node voltage at node b, connect the red probe of the voltmeter at node b and connect the black probe at the reference node, node c.

The node voltages in Figure 4.2-1*c* can be represented as v_{ac} and v_{bc}, but it is conventional to drop the subscript c and refer to these as v_a and v_b. Notice that the node voltage at the reference node is $v_{cc} = v_c = 0$ V, since a voltmeter measuring the node voltage at the reference node would have both probes connected to the same point.

One of the standard methods for analyzing an electric circuit is to write and solve a set of simultaneous equations called the node equations. The unknown variables in the node equations are the node voltages of the circuit. We determine the values of the node voltages by solving the node equations.

To write a set of node equations, we do two things:

1. Express element currents as functions of the node voltages.
2. Apply Kirchhoff's current law (KCL) at each of the nodes of the circuit, except for the reference node.

Consider the problem of expressing element currents as functions of the node voltages. Although our goal is to express element *currents* as functions of the node voltages, we begin by expressing element *voltages* as functions of the node voltages. Figure 4.2-2 shows how this is done. The voltmeters in Figure 4.2-2 measure the node voltages, v_1 and v_2, at the nodes of the circuit element. The element voltage has been labeled as v_a. Applying Kirchhoff's voltage law to the loop shown in Figure 4.2-2 gives

$$v_a = v_1 - v_2$$

This equation expresses the element voltage, v_a, as a function of the node voltages, v_1 and v_2. (There is an easy way to remember this equation. Notice the reference polarity of the element

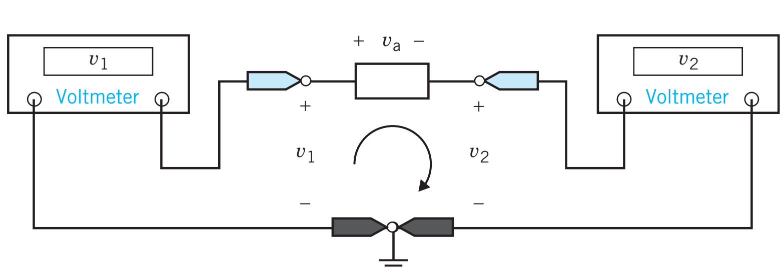

FIGURE 4.2-2 Node voltages, v_1 and v_2, and element voltage, v_a, of a circuit element.

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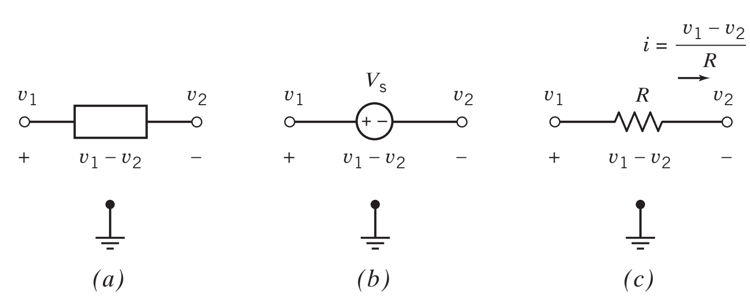

FIGURE 4.2-3 Node voltages, v_1 and v_2, and element voltage, $v_1 - v_2$, of a (*a*) generic circuit element, (*b*) voltage source, and (*c*) resistor.

voltage, v_a. The element voltage is equal to the node voltage at the node near the + of the reference polarity minus the node voltage at the node near the − of the reference polarity.)

Now consider Figure 4.2-3. In Figure 4.2-3*a*, we use what we have learned to express the voltage of a circuit element as a function of node voltages. The circuit element in Figure 4.2-3*a* could be anything: a resistor, a current source, a dependent voltage source, and so on. In Figures 4.2-3*b*, *c*, we consider specific types of circuit element. In Figure 4.2-3*b* the circuit element is a voltage source. The element voltage has been represented twice, once as the voltage source voltage, V_s, and once as a function of the node voltages, $v_1 - v_2$. Noticing that the reference polarities for V_s and $v_1 - v_2$ are the same (both + on the left), we write

$$V_s = v_1 - v_2$$

This is an important result. Whenever we have a voltage source connected between two nodes of a circuit, we can express the voltage source voltage, V_s, as a function of the node voltages, v_1 and v_2.

Frequently we know the value of the voltage source voltage. For example, suppose that $V_s = 12$ V. Then

$$12 = v_1 - v_2$$

This equation relates the values of two of the node voltages.

Next consider Figure 4.2-3*c*. In Figure 4.2-3*c* the circuit element is a resistor. We will use Ohm's law to express the resistor current, *i*, as a function of the node voltages. First, we express the resistor voltage as a function of the node voltages, $v_1 - v_2$. Noticing that the resistor voltage, $v_1 - v_2$, and the current, *i*, adhere to the passive convention, we use Ohm's law to write

$$i = \frac{v_1 - v_2}{R}$$

Frequently we know the value of the resistance. For example when $R = 8\ \Omega$, this equation becomes

$$i = \frac{v_1 - v_2}{8}$$

This equation expresses the resistor current, *i*, as a function of the node voltages, v_1 and v_2.

Next let's write node equations to represent the circuit shown in Figure 4.2-4*a*. The input to this circuit is the current source current, i_s. To write node equations, we will first express the resistor currents as functions of the node voltages and then apply Kirchhoff's current law at nodes a and b. The resistor voltages are expressed as functions of the node voltages in Figure 4.2-4*b*, and then the resistor currents are expressed as functions of the node voltages in Figure 4.2-4*c*.

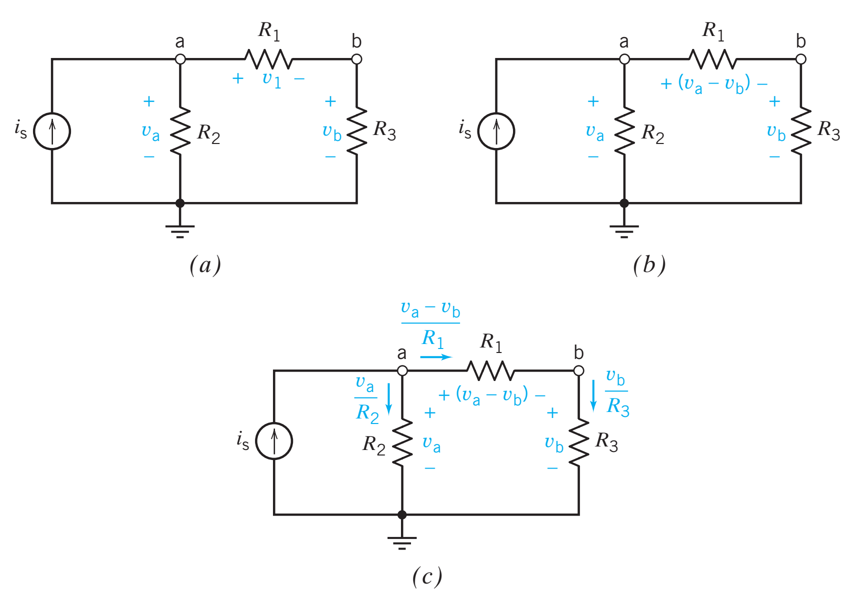

FIGURE 4.2-4 (*a*) A circuit with three resistors. (*b*) The resistor voltages expressed as functions of the node voltages. (*c*) The resistor currents expressed as functions of the node voltages.

The node equations representing the circuit in Figure 4.2-4 are obtained by applying Kirchhoff's current law at nodes a and b. Using KCL at node a gives

$$i_s = \frac{v_a}{R_2} + \frac{v_a - v_b}{R_1} \tag{4.2-1}$$

Similarly, the KCL equation at node b is

$$\frac{v_a - v_b}{R_1} = \frac{v_b}{R_3} \tag{4.2-2}$$

If $R_1 = 1\ \Omega$, $R_2 = R_3 = 0.5\ \Omega$, and $i_s = 4$ A, Eqs. 4.2-1 and 4.2-2 may be rewritten as

$$4 = \frac{v_a - v_b}{1} + \frac{v_a}{0.5} \tag{4.2-3}$$

$$\frac{v_a - v_b}{1} = \frac{v_b}{0.5} \tag{4.2-4}$$

Solving Eq. 4.2-4 for v_b gives

$$v_b = \frac{v_a}{3} \tag{4.2-5}$$

Substituting Eq. 4.2-5 into Eq. 4.2-3 gives

$$4 = v_a - \frac{v_a}{3} + 2v_a = \frac{8}{3} v_a \tag{4.2-6}$$

Solving Eq. 4.2-6 for v_a gives

$$v_a = \frac{3}{2}\ \text{V}$$

Finally, Eq. 4.2-5 gives

$$v_b = \frac{1}{2}\ \text{V}$$

Thus, the node voltages of this circuit are

$$v_a = \frac{3}{2}\text{ V} \quad\text{and}\quad v_b = \frac{1}{2}\text{ V}$$

EXAMPLE 4.2-1 *Node Equations*

Determine the value of the resistance R in the circuit shown in Figure 4.2-5*a*.

Solution

Let v_a denote the node voltage at node a and v_b denote the node voltage at node b. The voltmeter in Figure 4.2-5 measures the value of the node voltage at node b, v_b. In Figure 4.2-5*b*, the resistor currents are expressed as functions of the node voltages. Apply KCL at node a to obtain

$$4 + \frac{v_a}{10} + \frac{v_a - v_b}{5} = 0$$

Using $v_b = 5$ V, gives

$$4 + \frac{v_a}{10} + \frac{v_a - 5}{5} = 0$$

Solving for v_a, we get

$$v_a = -10\text{ V}$$

Next, apply KCL at node b to obtain

$$-\left(\frac{v_a - v_b}{5}\right) + \frac{v_b}{R} - 4 = 0$$

Using $v_a = -10$ V and $v_b = 5$ V gives

$$-\left(\frac{-10 - 5}{5}\right) + \frac{5}{R} - 4 = 0$$

Finally, solving for R gives

$$R = 5\ \Omega$$

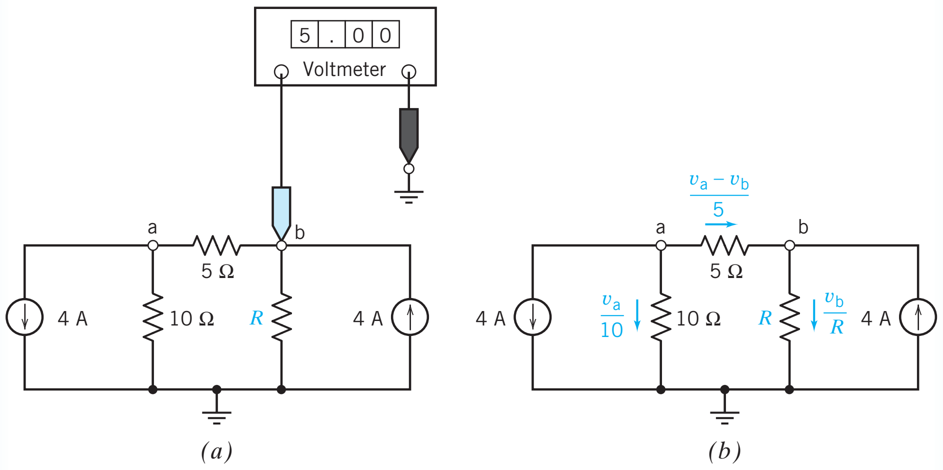

FIGURE 4.2-5 (*a*) The circuit for Example 4.2-1. (*b*) The circuit after the resistor currents are expressed as functions of the node voltages.

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EXAMPLE 4.2-2 *Node Equations*

Obtain the node equations for the circuit in Figure 4.2-6.

Solution

Let v_a denote the node voltage at node a, v_b denote the node voltage at node b, and v_c denote the node voltage at node c. Apply KCL at node a to obtain

$$-\left(\frac{v_a - v_c}{R_1}\right) + i_1 - \left(\frac{v_a - v_c}{R_2}\right) + i_2 - \left(\frac{v_a - v_b}{R_5}\right) = 0$$

Separate the terms of this equation that involve v_a from the terms that involve v_b and the terms that involve v_c to obtain

$$\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_5}\right)v_a - \left(\frac{1}{R_5}\right)v_b - \left(\frac{1}{R_1} + \frac{1}{R_2}\right)v_c = i_1 + i_2$$

There is a pattern in the node equations of circuits that contain only resistors and current sources. In the node equation at node a, the coefficient of v_a is the sum of the reciprocals of the resistances of all resistors connected to node a. The coefficient of v_b is minus the sum of the reciprocals of the resistances of all resistors connected between node b and node a. The coefficient of v_c is minus the sum of the reciprocals of the resistances of all resistors connected between node c and node a. The right-hand side of this equation is the algebraic sum of current source currents directed into node a.

Apply KCL at node b to obtain

$$-i_2 + \left(\frac{v_a - v_b}{R_5}\right) - \left(\frac{v_b - v_c}{R_3}\right) - \left(\frac{v_b}{R_4}\right) + i_3 = 0$$

Separate the terms of this equation that involve v_a from the terms that involve v_b and the terms that involve v_c to obtain

$$-\left(\frac{1}{R_5}\right)v_a + \left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right)v_b - \left(\frac{1}{R_3}\right)v_c = i_3 - i_2$$

As expected, this node equation adheres to the pattern for node equations of circuits that contain only resistors and current sources. In the node equation at node b, the coefficient of v_b is the sum of the reciprocals of the resistances of all resistors connected to node b. The coefficient of v_a is minus the sum of the reciprocals of the resistances of all resistors connected between node a and node b. The coefficient of v_c is minus the sum of the reciprocals of the resistances of all resistors connected between node c and node b. The right-hand side of this equation is the algebraic sum of current source currents directed into node b.

Finally, use the pattern for the node equations of circuits that contain only resistors and current sources to obtain the node equation at node c:

$$-\left(\frac{1}{R_1} + \frac{1}{R_2}\right)v_a - \left(\frac{1}{R_3}\right)v_b + \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_6}\right)v_c = -i_1$$

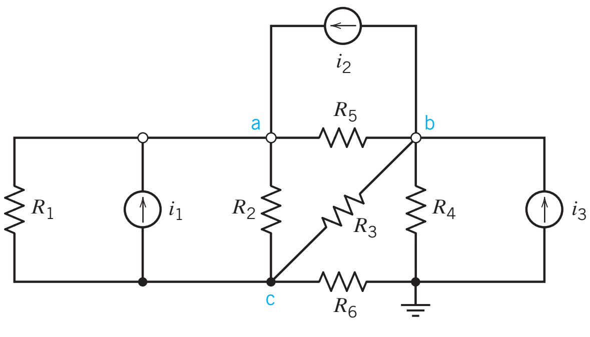

FIGURE 4.2-6
The circuit for Example 4.2-2.

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EXAMPLE 4.2-3 *Node Equations*

Determine the node voltages for the circuit in Figure 4.2-6 when $i_1 = 1$ A, $i_2 = 2$ A, $i_3 = 3$ A, $R_1 = 5\ \Omega$, $R_2 = 2\ \Omega$, $R_3 = 10\ \Omega$, $R_4 = 4\ \Omega$, $R_5 = 5\ \Omega$, and $R_6 = 2\ \Omega$.

Solution

The node equations are

$$\left(\frac{1}{5}+\frac{1}{2}+\frac{1}{5}\right)v_a - \left(\frac{1}{5}\right)v_b - \left(\frac{1}{5}+\frac{1}{2}\right)v_c = 1+2$$
$$-\left(\frac{1}{5}\right)v_a + \left(\frac{1}{10}+\frac{1}{5}+\frac{1}{4}\right)v_b - \left(\frac{1}{10}\right)v_c = -2+3$$
$$-\left(\frac{1}{5}+\frac{1}{2}\right)v_a - \left(\frac{1}{10}\right)v_b + \left(\frac{1}{5}+\frac{1}{2}+\frac{1}{10}+\frac{1}{2}\right)v_c = -1$$

or

$$0.9v_a - 0.2v_b - 0.7v_c = 3$$
$$-0.2v_a + 0.55v_b - 0.1v_c = 1$$
$$-0.7v_a - 0.1v_b + 1.3v_c = -1$$

The node equations can be written using matrices as

$$\begin{bmatrix} 0.9 & -0.2 & -0.7 \\ -0.2 & 0.55 & -0.1 \\ -0.7 & -0.1 & 1.3 \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix}$$

This matrix equation can be solved using Cramer's rule (see Appendix A). First calculate

$$\Delta = \begin{vmatrix} 0.9 & -0.2 & -0.7 \\ -0.2 & 0.55 & -0.1 \\ -0.7 & -0.1 & 1.3 \end{vmatrix} = 0.2850$$

Next, we find

$$v_a = \frac{\begin{vmatrix} 3 & -0.2 & -0.7 \\ 1 & 0.55 & -0.1 \\ -1 & -0.1 & 1.3 \end{vmatrix}}{\Delta} = 7.1579, \quad v_b = \frac{\begin{vmatrix} 0.9 & 3 & -0.7 \\ -0.2 & 1 & -0.1 \\ -0.7 & -1 & 1.3 \end{vmatrix}}{\Delta} = 5.0526 \quad \text{and}$$

$$v_c = \frac{\begin{vmatrix} 0.9 & -0.2 & 3 \\ -0.2 & 0.55 & 1 \\ -0.7 & -0.1 & -1 \end{vmatrix}}{\Delta} = 3.4737$$

Exercise 4.2-1 Determine the node voltages, v_a and v_b, for the circuit of Figure E 4.2-1.
Answer: $v_a = 3$ V and $v_b = 11$ V

Exercise 4.2-2 Determine the node voltages, v_a and v_b, for the circuit of Figure E 4.2-2.
Answer: $v_a = -4/3$ V and $v_b = 4$ V

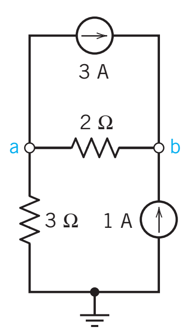

FIGURE E 4.2-1

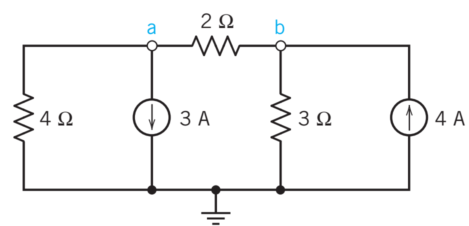

FIGURE E 4.2-2

4.3 Node Voltage Analysis of Circuits with Current and Voltage Sources

In the preceding section we determined the node voltages of circuits with independent current sources only. In this section we consider circuits with both independent current and voltage sources.

First we consider the circuit with a voltage source between ground and one of the other nodes. Since we are free to select the reference node, this particular arrangement is easily achieved. Such a circuit is shown in Figure 4.3-1. We immediately note that the source is connected between terminal a and ground, and therefore

$$v_a = v_s$$

Thus, v_a is known and only v_b is unknown. We write the KCL equation at node b to obtain

$$i_s = \frac{v_b}{R_3} + \frac{v_b - v_a}{R_2}$$

However, $v_a = v_s$. Therefore

$$i_s = \frac{v_b}{R_3} + \frac{v_b - v_s}{R_2}$$

Then, solving for the unknown node voltage v_b, we get

$$v_b = \frac{R_2 R_3 i_s + R_3 v_s}{R_2 + R_3}$$

Next, let us consider the circuit of Figure 4.3-2, which includes a voltage source between two nodes. Since the source voltage is known, use KVL to obtain

$$v_a - v_b = v_s$$

or

$$v_a = v_s + v_b$$

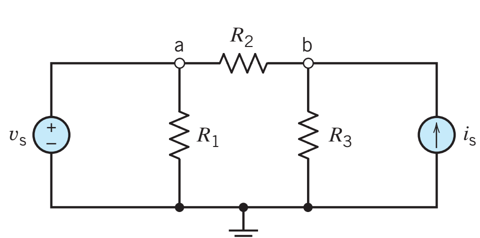

FIGURE 4.3-1 Circuit with an independent voltage source and an independent current source.

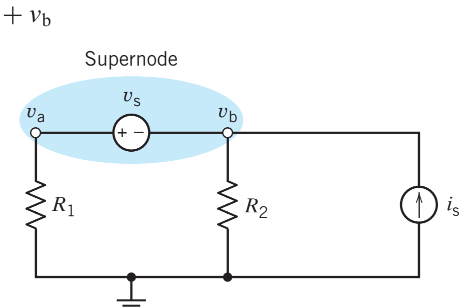

FIGURE 4.3-2 Circuit with a supernode that incorporates v_a and v_b.

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Table 4.3-1 Node Voltage Analysis Methods with a Voltage Source

CASE	METHOD
1. The voltage source connects node q and the reference node (ground).	Set v_q equal to the source voltage accounting for the polarities and proceed to write the KCL at the remaining nodes.
2. The voltage source lies between two nodes, a and b.	Create a supernode that incorporates a and b and equate the sum of all the currents into the supernode to zero.

To account for the fact that the source voltage is known, we consider both node a and node b as part of one larger node represented by the shaded ellipse shown in Figure 4.3-2. We require a larger node since v_a and v_b are dependent. This larger node is often called a *supernode* or a *generalized node*. KCL says that the algebraic sum of the currents entering a supernode is zero. That means that we apply KCL to a supernode in the same way that we apply KCL to a node.

> A **supernode** consists of two nodes connected by an independent or a dependent voltage source.

We then can write the KCL equation at the supernode as

$$\frac{v_a}{R_1} + \frac{v_b}{R_2} = i_s$$

However, since $v_a = v_s + v_b$, we have

$$\frac{v_s + v_b}{R_1} + \frac{v_b}{R_2} = i_s$$

Then, solving for the unknown node voltage v_b, we get

$$v_b = \frac{R_1 R_2 i_s - R_2 v_s}{R_1 + R_2}$$

We can now compile a summary of both methods of dealing with independent voltage sources in a circuit we wish to solve by node voltage methods, as recorded in Table 4.3-1.

EXAMPLE 4.3-1 *Node Equations for a Circuit Containing Voltage Sources*

Determine the node voltages for the circuit shown in Figure 4.3-3.

Solution

The methods summarized in Table 4.3-1 are exemplified in this solution. The 4-V voltage source connected to node a exemplifies method 1. The 8-V source between nodes b and c exemplifies method 2.

Using method 1 for the 4-V source, we note that

$$v_a = -4 \text{ V}$$

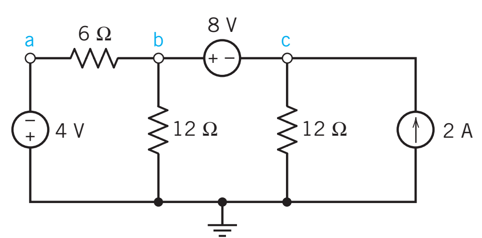

FIGURE 4.3-3 A circuit containing two voltage sources, only one of which is connected to the reference node.

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Using method 2 for the 8-V source, we have a supernode at nodes b and c. The node voltages at nodes b and c are related by

$$v_b = v_c + 8$$

Writing a KCL equation for the supernode, we have

$$\frac{v_b - v_a}{6} + \frac{v_b}{12} + \frac{v_c}{12} = 2$$

or

$$3\,v_b + v_c = 24 + 2\,v_a$$

Using $v_a = -4$ V and $v_b = v_c + 8$ to eliminate v_a and v_b, we have

$$3(v_c + 8) + v_c = 24 + 2(-4)$$

Solving this equation for v_c, we get

$$v_c = -2\text{ V}$$

Now we calculate v_b to be

$$v_b = v_c + 8 = -2 + 8 = 6\text{ V}$$

EXAMPLE 4.3-2 *Supernodes*

Determine the values of the node voltages, v_a and v_b, for the circuit shown in Figure 4.3-4.

Solution

We can write the first node equation by considering the voltage source. The voltage source voltage is related to the node voltages by

$$v_b - v_a = 12 \quad \Rightarrow \quad v_b = v_a + 12$$

In order to write the second node equation, we must decide what to do about the voltage source current. (Notice that there is no easy way to express the voltage source current in terms of the node voltages.) In this example, we illustrate two methods of writing the second node equation.

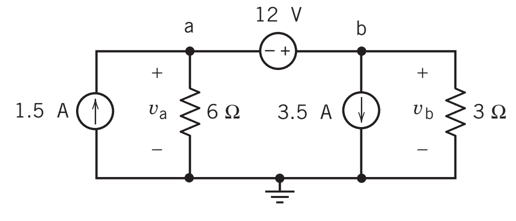

FIGURE 4.3-4
The circuit for Example 4.3-2.

Method 1: Assign a name to the voltage source current. Apply KCL at both of the voltage source nodes. Eliminate the voltage source current from the KCL equations.

Figure 4.3-5 shows the circuit after labeling the voltage source current. The KCL equation at node a is

$$1.5 + i = \frac{v_a}{6}$$

The KCL equation at node b is

$$i + 3.5 + \frac{v_b}{3} = 0$$

Combining these two equations gives

$$1.5 - \left(3.5 + \frac{v_b}{3}\right) = \frac{v_a}{6} \quad \Rightarrow \quad -2.0 = \frac{v_a}{6} + \frac{v_b}{3}$$

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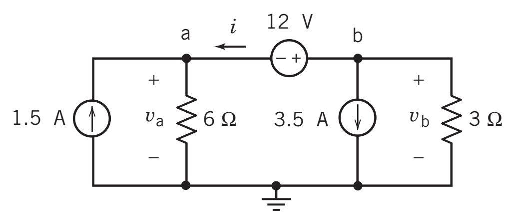

FIGURE 4.3-5
Method 1 for Example 4.3-2.

Method 2: Apply KCL to the supernode corresponding to the voltage source. Shown in Figure 4.3-6, this supernode separates the voltage source and its nodes from the rest of the circuit. (In this small circuit, the rest of the circuit is just the reference node.)

Apply KCL to the supernode to get

$$1.5 = \frac{v_a}{6} + 3.5 + \frac{v_b}{3} \quad \Rightarrow \quad -2.0 = \frac{v_a}{6} + \frac{v_b}{3}$$

This is the same equation that was obtained using method 1. Applying KCL to the supernode is a shortcut for doing three things:

1. Labeling the voltage source current as *i*
2. Applying KCL at both nodes of the voltage source
3. Eliminating i from the KCL equations

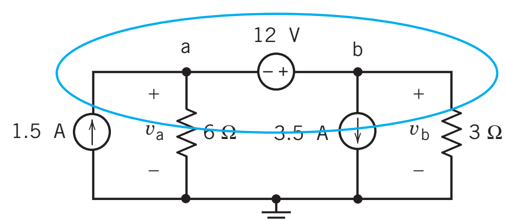

FIGURE 4.3-6
Method 2 for Example 4.3-2.

In summary, the node equations are

$$v_b - v_a = 12$$

and

$$\frac{v_a}{6} + \frac{v_b}{3} = -2.0$$

Solving the node equations gives

$$v_a = -12\text{ V} \quad \text{and} \quad v_b = 0\text{ V}$$

(We might be surprised that v_b is 0 V, but it is easy to check that these values are correct by substituting them into the node equations.)

EXAMPLE 4.3-3 *Node Equations for a Circuit Containing Voltage Sources*

Determine the node voltages for the circuit shown in Figure 4.3-7.

Solution

We will calculate the node voltages of this circuit by writing a KCL equation for the supernode corresponding to the 10-V voltage source. First notice that

$$v_b = -12\text{ V}$$

and that

$$v_a = v_c + 10$$

Writing a KCL equation for the supernode, we have

$$\frac{v_a - v_b}{10} + 2 + \frac{v_c - v_b}{40} = 5$$

or

$$4\,v_a + v_c - 5\,v_b = 120$$

Using $v_a = v_c + 10$ and $v_b = -12$ to eliminate v_a and v_b, we have

$$4(v_c + 10) + v_c - 5(-12) = 120$$

Solving this equation for v_c we get

$$v_c = 4\text{ V}$$

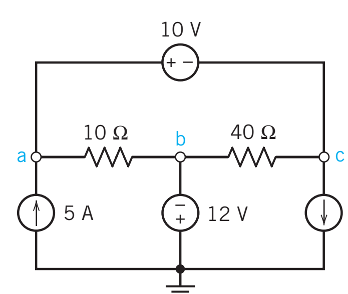

FIGURE 4.3-7 The circuit for Example 4.3-3.

Exercise 4.3-1 Find the node voltages for the circuit of Figure E 4.3-1.

Hint: Write a KCL equation for the supernode corresponding to the 10-V voltage source.

Answer: $2 + \dfrac{v_b + 10}{20} + \dfrac{v_b}{30} = 5 \quad \Rightarrow \quad v_b = 30\text{ V} \quad \text{and} \quad v_a = 40\text{ V}$

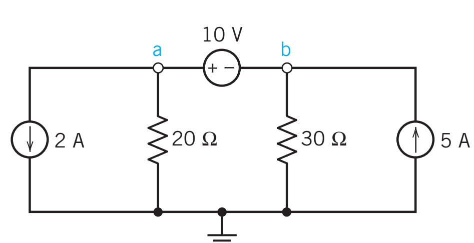

FIGURE E 4.3-1

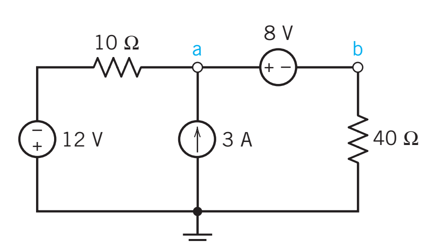

FIGURE E 4.3-2

Exercise 4.3-2 Find the voltages v_a and v_b for the circuit of Figure E 4.3-2.

Answer: $\dfrac{(v_b + 8) - (-12)}{10} + \dfrac{v_b}{40} = 3 \quad \Rightarrow \quad v_b = 8\text{ V} \quad \text{and} \quad v_a = 16\text{ V}$

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4.4 Node Voltage Analysis with Dependent Sources

When a circuit contains a dependent source, the controlling current or voltage of that dependent source must be expressed as a function of the node voltages.

It is then a simple matter to express the controlled current or voltage as a function of the node voltages. The node equations are then obtained using the techniques described in the previous two sections.

EXAMPLE 4.4-1 *Node Equations for a Circuit Containing a Dependent Source*

Determine the node voltages for the circuit shown in Figure 4.4-1.

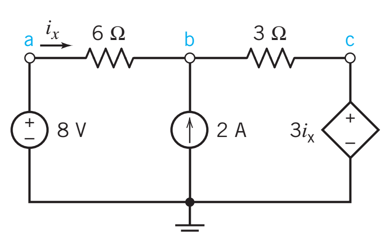

FIGURE 4.4-1
A circuit with a CCVS.

Solution

The controlling current of the dependent source is i_x. Our first task is to express this current as a function of the node voltages:

$$i_x = \frac{v_a - v_b}{6}$$

The value of the node voltage at node a is set by the 8-V voltage source to be

$$v_a = 8 \text{ V}$$

so

$$i_x = \frac{8 - v_b}{6}$$

The node voltage at node c is equal to the voltage of the dependent source, so

$$v_c = 3i_x = 3\left(\frac{8 - v_b}{6}\right) = 4 - \frac{v_b}{2} \quad (4.4\text{-}1)$$

Next, apply KCL at node b to get

$$\frac{8 - v_b}{6} + 2 = \frac{v_b - v_c}{3} \quad (4.4\text{-}2)$$

Using Eq. 4.4-1 to eliminate v_c from Eq. 4.4-2 gives

$$\frac{8 - v_b}{6} + 2 = \frac{v_b - \left(4 - \frac{v_b}{2}\right)}{3} = \frac{v_b}{2} - \frac{4}{3}$$

Solving for v_b gives

$$v_b = 7 \text{ V}$$

Then

$$v_c = 4 - \frac{v_b}{2} = \frac{1}{2} \text{ V}$$

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EXAMPLE 4.4-2

Determine the node voltages for the circuit shown in Figure 4.4-2.

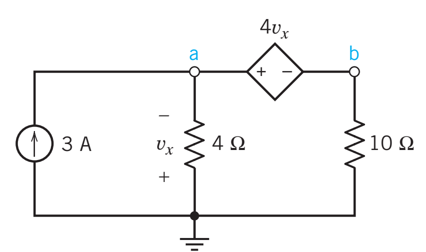

FIGURE 4.4-2
A circuit with a VCVS.

Solution

The controlling voltage of the dependent source is v_x. Our first task is to express this voltage as a function of the node voltages:

$$v_x = -v_a$$

The difference between the node voltages at nodes a and b is set by voltage of the dependent source:

$$v_a - v_b = 4\, v_x = 4(-v_a) = -4\, v_a$$

Simplifying this equation gives

$$v_b = 5\, v_a \tag{4.4-3}$$

Applying KCL to the supernode corresponding to the dependent voltage source gives

$$3 = \frac{v_a}{4} + \frac{v_b}{10} \tag{4.4-4}$$

Using Eq. 4.4-3 to eliminate v_b from Eq. 4.4-4 gives

$$3 = \frac{v_a}{4} + \frac{5v_a}{10} = \frac{3}{4}\, v_a$$

Solving for v_a, we get

$$v_a = 4\ \text{V}$$

Finally,

$$v_b = 5\, v_a = 20\ \text{V}$$

EXAMPLE 4.4-3

Determine the node voltages corresponding to nodes a and b for the circuit shown in Figure 4.4-3.

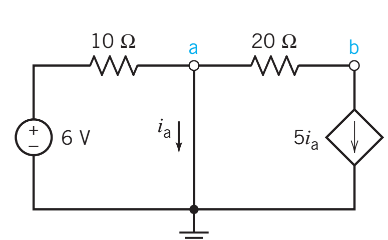

FIGURE 4.4-3
A circuit with a CCCS.

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Solution

The controlling current of the dependent source is i_a. Our first task is to express this current as a function of the node voltages. Apply KCL at node a to get

$$\frac{6 - v_a}{10} = i_a + \frac{v_a - v_b}{20}$$

Node a is connected to the reference node by a short circuit, so $v_a = 0$ V. Substituting this value of v_a into the above equation and simplifying gives

$$i_a = \frac{12 + v_b}{20} \tag{4.4-5}$$

Next, apply KCL at node b to get

$$\frac{0 - v_b}{20} = 5\, i_a \tag{4.4-6}$$

Using Eq. 4.4-5 to eliminate i_a from Eq. 4.4-6 gives

$$\frac{0 - v_b}{20} = 5\left(\frac{12 + v_b}{20}\right)$$

Solving for v_b gives

$$v_b = -10 \text{ V}$$

Exercise 4.4-1 Find the node voltage v_b for the circuit shown in Figure E 4.4-2.

Hint: Apply KCL at node a to express i_a as a function of the node voltages. Substitute the result into $v_b = 4i_a$ and solve for v_b.

Answer: $-\frac{6}{8} + \frac{v_b}{4} - \frac{v_b}{12} = 0 \Rightarrow v_b = 4.5$ V

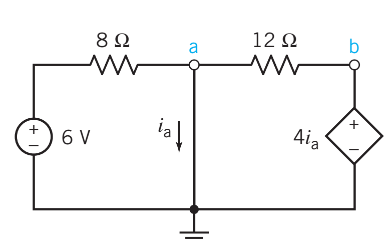

FIGURE E 4.4-1 A circuit with a CCVS.

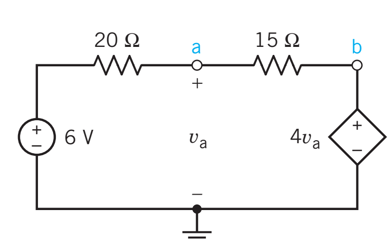

FIGURE E 4.4-2 A circuit with a VCVS.

Exercise 4.4-2 Find the node voltages for the circuit shown in Figure E 4.4-2.

Hint: The controlling voltage of the dependent source is a node voltage, so it is already expressed as a function of the node voltages. Apply KCL at node a.

Answer: $\frac{v_a - 6}{20} + \frac{v_a - 4v_a}{15} = 0 \Rightarrow v_a = -2$ V

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4.5 Mesh Current Analysis with Independent Voltage Sources

In this and succeeding sections, we consider the analysis of circuits using Kirchhoff's voltage law (KVL) around a closed path. A *closed path* or a *loop* is drawn by starting at a node and tracing a path such that we return to the original node without passing an intermediate node more than once.

A mesh is a special case of a loop.

> A *mesh* is a loop that does not contain any other loops within it.

Mesh current analysis is applicable only to planar networks. A planar circuit is one that can be drawn on a plane, without crossovers. An example of a nonplanar circuit is shown in Figure 4.5-1, where the crossover is identified and cannot be removed by redrawing the circuit. For planar networks, the meshes in the network look like "windows." There are four meshes in the circuit shown in Figure 4.5-2. They are identified as M_i. Mesh 2 contains the elements R_3, R_4, and R_5. Note that the resistor R_3 is common to both mesh 1 and mesh 2.

We define a mesh current as the current that flows through the elements constituting the mesh. Figure 4.5-3*a* shows a circuit having two meshes with the mesh currents labeled as i_1 and i_2. We will use the convention of a mesh current flowing clockwise as shown in Figure 4.5-3*a*. In Figure 4.5-3*b*, ammeters have been inserted into the meshes to measure the mesh currents.

One of the standard methods for analyzing an electric circuit is to write and solve a set of simultaneous equations called the mesh equations. The unknown variables in the mesh equations are the mesh currents of the circuit. We determine the values of the mesh currents by solving the mesh equations.

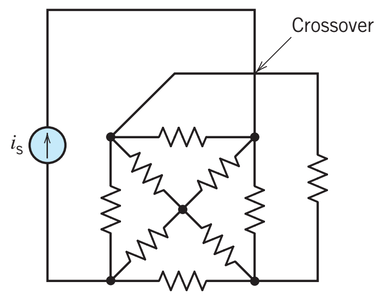

FIGURE 4.5-1
Nonplanar circuit with a crossover.

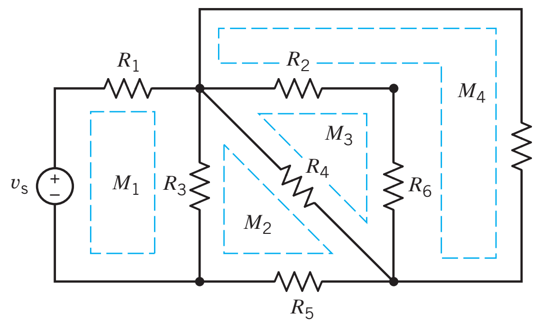

FIGURE 4.5-2
Circuit with four meshes. Each mesh is identified by dashed lines.

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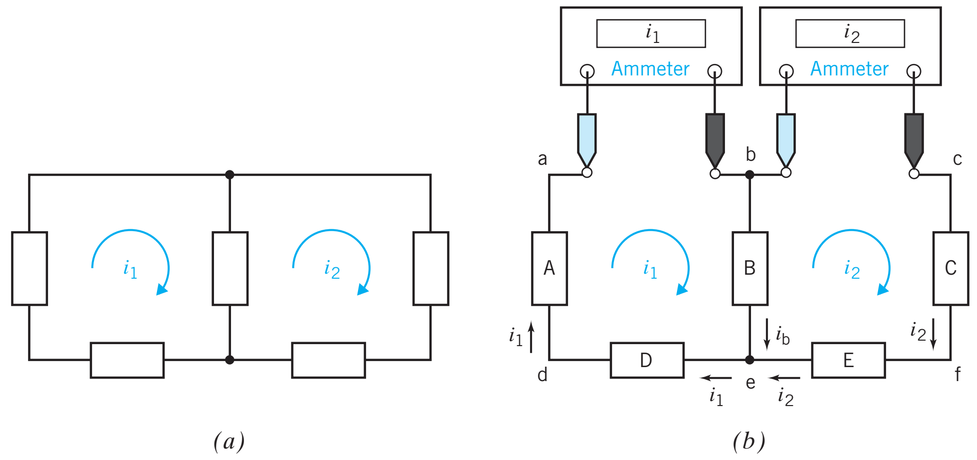

FIGURE 4.5-3 (*a*) A circuit with two meshes. (*b*) Inserting ammeters to measure the mesh currents.

To write a set of mesh equations, we do two things:

1. Express element voltages as functions of the mesh currents.
2. Apply Kirchhoff's voltage law (KVL) to each of the meshes of the circuit.

Consider the problem of expressing element voltages as functions of the mesh currents. Although our goal is to express element *voltages* as functions of the mesh currents, we begin by expressing element *currents* as functions of the mesh currents. Figure 4.5-3*b* shows how this is done. The ammeters in Figure 4.5-3*b* measure the mesh currents, i_1 and i_2. Elements C and E are in the right mesh but not in the left mesh. Apply Kirchhoff's current law at node c and then at node f to see that the currents in elements C and E are equal to the mesh current of the right mesh, i_2, as shown in Figure 4.5-3*b*. Similarly, elements A and D are only in the left mesh. The currents in elements A and D are equal to the mesh current of the left mesh, i_1, as shown in Figure 4.5-3*b*.

Element B is in both meshes. The current of element B has been labeled as i_b. Applying Kirchhoff's current law at node b in Figure 4.5-3*b* gives

$$i_b = i_1 - i_2$$

This equation expresses the element current, i_b, as a function of the mesh currents, i_1 and i_2.

Figure 4.5-4*a* shows a circuit element that is in two meshes. The current of the circuit element is expressed as a function of the mesh currents of the two meshes. The circuit element in Figure 4.5-4*a* could be anything: a resistor, a current source, a dependent voltage source, and so on. In Figures 4.5-4*b*, *c*, we consider specific types of circuit element. In Figure 4.5-4*b*, the circuit element is a current source. The element current has been represented twice, once as the current source current, 3 A, and once as a function of the mesh currents, $i_1 - i_2$. Noticing that the reference directions for 3 A and $i_1 - i_2$ are different (one points up, the other points down), we write

$$-3 = i_1 - i_2$$

This equation relates the values of two of the mesh currents.

Next consider Figure 4.5-4*c*. In Figure 4.5-4*c*, the circuit element is a resistor. We will use Ohm's law to express the resistor voltage, v, as functions of the mesh currents. First, we express

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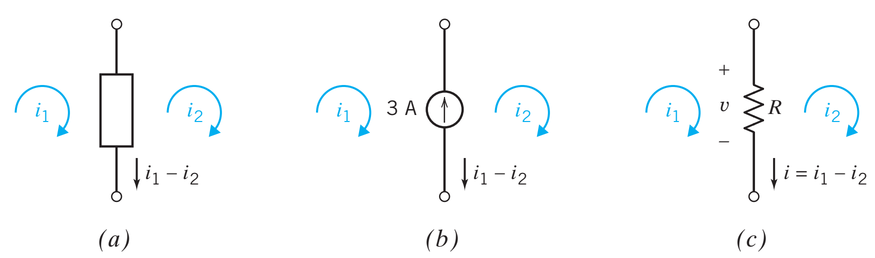

FIGURE 4.5-4 Mesh currents, i_1 and i_2, and element current, $i_1 - i_2$, of a (*a*) generic circuit element, (*b*) current source, and (*c*) resistor.

the resistor current as a function of the mesh currents, $i_1 - i_2$. Noticing that the resistor current, $i_1 - i_2$, and the voltage, v, adhere to the passive convention, we use Ohm's law to write

$$v = R(i_1 - i_2)$$

Frequently we know the value of the resistance. For example, when $R = 8\ \Omega$, this equation becomes

$$v = 8(i_1 - i_2)$$

This equation expresses the resistor voltage, v, as a function of the mesh currents, i_1 and i_2.

Next, let's write mesh equations to represent the circuit shown in Figure 4.5-5*a*. The input to this circuit is the voltage source voltage, v_s. To write mesh equations, we will first express the resistor voltages as functions of the mesh currents and then apply Kirchhoff's voltage law to the meshes. The resistor currents are expressed as functions of the mesh currents in Figure 4.5-5*b*, and then the resistor voltages are expressed as functions of the mesh currents in Figure 4.5-5*c*.

We may use Kirchhoff's voltage law around each mesh. We will use the following convention for obtaining the algebraic sum of voltages around a mesh. We will move around the mesh in the clockwise direction. If we encounter the + sign of the voltage reference polarity of an element voltage before the − sign, we add that voltage. Conversely, if we encounter the − of

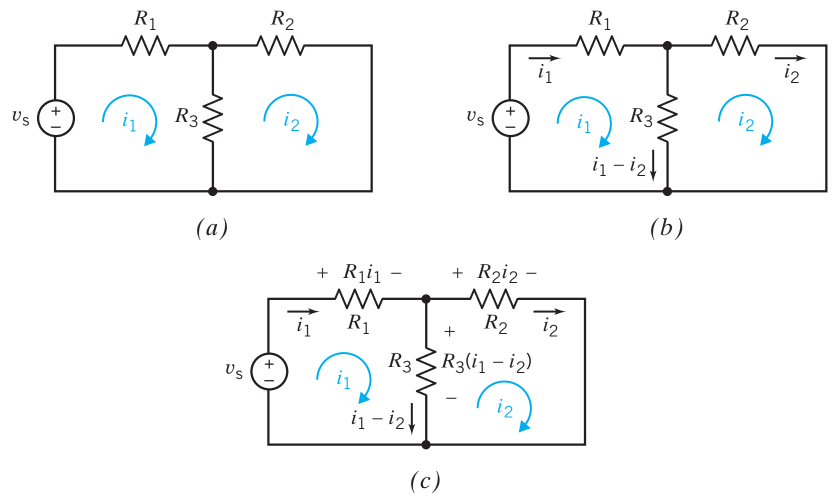

FIGURE 4.5-5 (*a*) A circuit. (*b*) The resistor currents expressed as functions of the mesh currents. (*c*) The resistor voltages expressed as functions of the mesh currents.

the voltage reference polarity of an element voltage before the + sign, we subtract that voltage. Thus, for the circuit of Figure 4.5-5*c*, we have

$$\text{mesh 1:} \quad -v_s + R_1 i_1 + R_3(i_1 - i_2) = 0 \tag{4.5-1}$$

$$\text{mesh 2:} \quad -R_3(i_1 - i_2) + R_2 i_2 = 0 \tag{4.5-2}$$

Note that the voltage across R_3 in mesh 1 is determined from Ohm's law, where

$$v = R_3 i_a = R_3(i_1 - i_2)$$

where i_a is the actual element current flowing downward through R_3.

Equations 4.5-1 and 4.5-2 will enable us to determine the two mesh currents i_1 and i_2. Rewriting the two equations, we have

$$i_1(R_1 + R_3) - i_2 R_3 = v_s$$

and

$$-i_1 R_3 + i_2(R_3 + R_2) = 0$$

If $R_1 = R_2 = R_3 = 1\ \Omega$, we have

$$2i_1 - i_2 = v_s$$

and

$$-i_1 + 2i_2 = 0$$

Add twice the first equation to the second equation, obtaining $3i_1 = 2v_s$. Then we have

$$i_1 = \frac{2\, v_s}{3} \quad \text{and} \quad i_2 = \frac{v_s}{3}$$

Thus, we have obtained two independent mesh current equations that are readily solved for the two unknowns. If we have N meshes and write N mesh equations in terms of N mesh currents, we can obtain N independent mesh equations. This set of N equations is independent and thus guarantees a solution for the N mesh currents.

A circuit that contains only independent voltage sources and resistors results in a specific format of equations that can readily be obtained. Consider a circuit with three meshes, as shown in Figure 4.5-6. Assign the clockwise direction to all of the mesh currents. Using KVL, we obtain the three mesh equations

$$\text{mesh 1:} \quad -v_s + R_1 i_1 + R_4(i_1 - i_2) = 0$$
$$\text{mesh 2:} \quad R_2 i_2 + R_5(i_2 - i_3) + R_4(i_2 - i_1) = 0$$
$$\text{mesh 3:} \quad R_5(i_3 - i_2) + R_3 i_3 + v_g = 0$$

These three mesh equations can be rewritten by collecting coefficients for each mesh current as

$$\text{mesh 1:} \quad (R_1 + R_4)i_1 - R_4 i_2 = v_s$$
$$\text{mesh 2:} \quad -R_4 i_1 + (R_4 + R_2 + R_5)i_2 - R_5 i_3 = 0$$
$$\text{mesh 3:} \quad -R_5 i_2 + (R_3 + R_5)i_3 = -v_g$$

Hence, we note that the coefficient of the mesh current i_1 for the first mesh is the sum of resistances in mesh 1 and the coefficient of the second mesh current is the negative of the

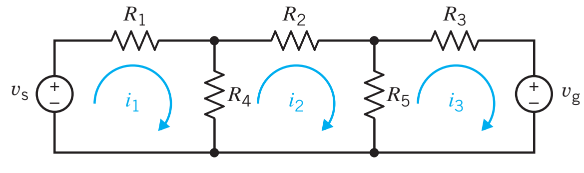

FIGURE 4.5-6 Circuit with three mesh currents and two voltage sources.

resistance common to meshes 1 and 2. In general, we state that for mesh current i_n, the equation for the nth mesh with independent voltage sources only is obtained as follows:

$$-\sum_{q=1}^{Q} R_k i_q + \sum_{j=1}^{P} R_j i_n = -\sum_{n=1}^{N} v_{sn} \tag{4.5-3}$$

That is, for mesh n we multiply i_n by the sum of all resistances R_j around the mesh. Then we add the terms due to the resistances in common with another mesh as the negative of the connecting resistance R_k, multiplied by the mesh current in the adjacent mesh i_q for all Q adjacent meshes. Finally, the independent voltage sources around the loop appear on the right side of the equation as the negative of the voltage sources encountered as we traverse the loop in the direction of the mesh current. Remember that the above result is obtained assuming all mesh currents flow clockwise.

The general matrix equation for the mesh current analysis for independent voltage sources present in a circuit is

$$\mathbf{R}\,\mathbf{i} = \mathbf{v}_s \tag{4.5-4}$$

where **R** is a symmetric matrix with a diagonal consisting of the sum of resistances in each mesh and the off-diagonal elements are the negative of the sum of the resistances common to two meshes. The matrix **i** consists of the mesh current as

$$\mathbf{i} = \begin{bmatrix} i_1 \\ i_2 \\ \cdot \\ \cdot \\ \cdot \\ i_N \end{bmatrix}$$

For N mesh currents, the source matrix $\mathbf{v}_s$ is

$$\mathbf{v}_s = \begin{bmatrix} v_{s1} \\ v_{s2} \\ \cdot \\ \cdot \\ \cdot \\ v_{sN} \end{bmatrix}$$

where v_{sj} is the algebraic sum of the voltages of the voltage sources in the jth mesh with the appropriate sign assigned to each voltage.

For the circuit of Figure 4.5-6 and the matrix Eq. 4.5-4, we have

$$\mathbf{R} = \begin{bmatrix} (R_1 + R_4) & -R_4 & 0 \\ -R_4 & (R_2 + R_4 + R_5) & -R_5 \\ 0 & -R_5 & (R_3 + R_5) \end{bmatrix}$$

Note that **R** is a symmetric matrix, as we expected.

Exercise 4.5-1 Determine the value of the voltage measured by the voltmeter in Figure E 4.5-1.

Answer: -1 V

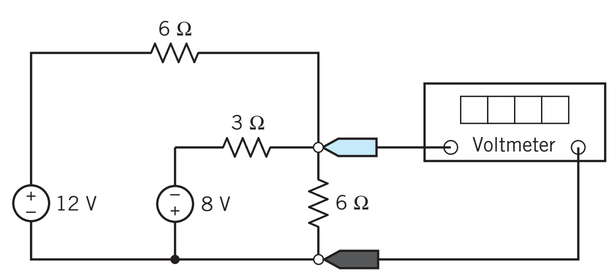

FIGURE E 4.5-1

4.6 Mesh Current Analysis with Current and Voltage Sources

Heretofore, we have considered only circuits with independent voltage sources for analysis by the mesh current method. If the circuit has an independent current source, as shown in Figure 4.6-1, we recognize that the second mesh current is equal to the negative of the current source current. We can then write

$$i_2 = -i_s$$

and we need only determine the first mesh current i_1. Writing KVL for the first mesh, we obtain

$$(R_1 + R_2)i_1 - R_2 i_2 = v_s$$

Since $i_2 = -i_s$, we have

$$i_1 = \frac{v_s - R_2 i_s}{R_1 + R_2} \qquad (4.6\text{-}1)$$

where i_s and v_s are sources of known magnitude.

If we encounter a circuit as shown in Figure 4.6-2, we have a current source i_s that has an unknown voltage v_{ab} across its terminals. We can readily note that

$$i_2 - i_1 = i_s \qquad (4.6\text{-}2)$$

by writing KCL at node a. The two mesh equations are

$$\text{mesh 1:} \quad R_1 i_1 + v_{ab} = v_s \qquad (4.6\text{-}3)$$

$$\text{mesh 2:} \quad (R_2 + R_3)i_2 - v_{ab} = 0 \qquad (4.6\text{-}4)$$

We note that if we add Eqs. 4.6-3 and 4.6-4, we eliminate v_{ab}, obtaining

$$R_1 i_1 + (R_2 + R_3)i_2 = v_s$$

However, since $i_2 = i_s + i_1$, we obtain

$$R_1 i_1 + (R_2 + R_3)(i_s + i_1) = v_s$$

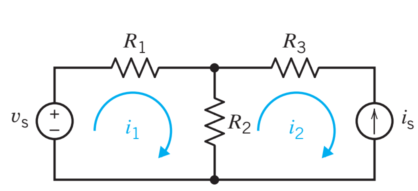

FIGURE 4.6-1 Circuit with an independent voltage source and an independent current source.

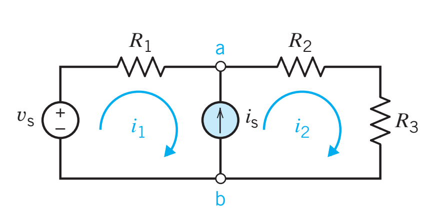

FIGURE 4.6-2 Circuit with an independent current source common to both meshes.

or

$$i_1 = \frac{v_s - (R_2 + R_3)i_s}{R_1 + R_2 + R_3} \tag{4.6-5}$$

Thus, we account for independent current sources by recording the relationship between the mesh currents and the current source current. If the current source influences *only one* mesh current, we write the equation that relates that mesh current to the current source current and write the KVL equations for the remaining meshes. If the current source influences two mesh currents, we write the KVL equation for both meshes, assuming a voltage v_{ab} across the terminals of the current source. Then, adding these two mesh equations, we obtain an equation independent of v_{ab}.

EXAMPLE 4.6-1 *Mesh Equations*

Consider the circuit of Figure 4.6-3 where $R_1 = R_2 = 1\ \Omega$ and $R_3 = 2\ \Omega$. Find the three mesh currents.

Solution

Since the 4-A source is only in mesh 1, we note that

$$i_1 = 4$$

For the 5-A source, we have

$$i_2 - i_3 = 5 \tag{4.6-6}$$

Writing KVL for mesh 2 and mesh 3, we obtain

$$\text{mesh 2:}\quad R_1(i_2 - i_1) + v_{ab} = 10 \tag{4.6-7}$$

$$\text{mesh 3:}\quad R_2(i_3 - i_1) + R_3 i_3 - v_{ab} = 0 \tag{4.6-8}$$

We substitute $i_1 = 4$ and add Eqs. 4.6-7 and 4.6-8 to obtain

$$R_1(i_2 - 4) + R_2(i_3 - 4) + R_3 i_3 = 10 \tag{4.6-9}$$

From Eq. 4.6-6, $i_2 = 5 + i_3$. Substituting into Eq. 4.6-9, we have

$$R_1(5 + i_3 - 4) + R_2(i_3 - 4) + R_3 i_3 = 10$$

Using the values for the resistors, we obtain

$$i_3 = \frac{13}{4}\ \text{A} \quad \text{and} \quad i_2 = 5 + i_3 = \frac{33}{4}\ \text{A}$$

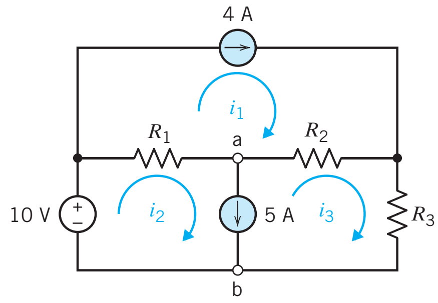

FIGURE 4.6-3 Circuit with two independent current sources.

Another technique for the mesh analysis method when a current source is common to two meshes involves the concept of a supermesh. A *supermesh* is one mesh created from two meshes that have a current source in common, as shown in Figure 4.6-4.

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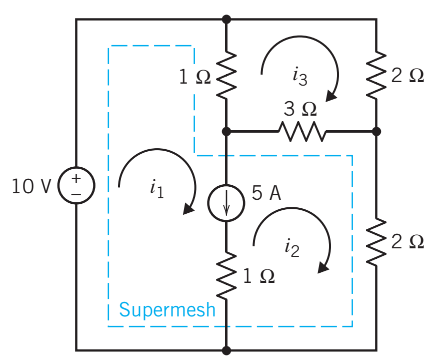

FIGURE 4.6-4
Circuit with a supermesh that incorporates mesh 1 and mesh 2. The supermesh is indicated by the dashed line.

A **supermesh** is one larger mesh created from two meshes that have an independent or dependent current source in common.

For example, consider the circuit of Figure 4.6-4. The 5-A current source is common to mesh 1 and mesh 2. The supermesh consists of the interior of mesh 1 and mesh 2. Writing KVL around the periphery of the supermesh shown by the dashed lines, we obtain

$$-10 + 1(i_1 - i_3) + 3(i_2 - i_3) + 2i_2 = 0$$

For mesh 3, we have

$$1(i_3 - i_1) + 2i_3 + 3(i_3 - i_2) = 0$$

Finally, the equation that relates the current source current to the mesh currents is

$$i_1 - i_2 = 5$$

Then the three equations may be reduced to

$$\begin{aligned} \text{supermesh:} \quad & 1i_1 + 5i_2 - 4i_3 = 10 \\ \text{mesh 3:} \quad & -1i_1 - 3i_2 + 6i_3 = 0 \\ \text{current source:} \quad & 1i_1 - 1i_2 = 5 \end{aligned}$$

Therefore, solving the three equations simultaneously, we find that $i_2 = 2.5$ A, $i_1 = 7.5$ A, and $i_3 = 2.5$ A.

The methods of mesh current analysis utilized when a current source is present are summarized in Table 4.6-1.

Table 4.6-1 Mesh Current Analysis Methods with a Current Source

CASE	METHOD
1. A current source appears on the periphery of only one mesh, *n*.	Equate the mesh current i_n to the current source current, accounting for the direction of the current source.
2. A current source is common to two meshes	A. Assume a voltage v_{ab} across the terminals of the current source, write the KVL equations for the two meshes, and add them to eliminate v_{ab} or B. Create a supermesh as the periphery of the two meshes and write one KVL equation around the periphery of the supermesh. In addition, write the constraining equation for the two mesh currents in terms of the current source.

EXAMPLE 4.6-2 *Supermeshes*

Determine the values of the mesh currents, i_1 and i_2, for the circuit shown in Figure 4.6-5.

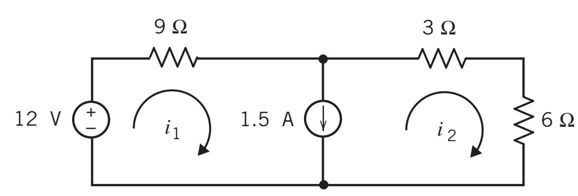

FIGURE 4.6-5
The circuit for Example 4.6-2.

Solution

We can write the first mesh equation by considering the current source. The current source current is related to the mesh currents at by

$$i_1 - i_2 = 1.5 \quad \Rightarrow \quad i_1 = i_2 + 1.5$$

In order to write the second mesh equation, we must decide what to do about the current source voltage. (Notice that there is no easy way to express the current source voltage in terms of the mesh currents.) In this example, we illustrate two methods of writing the second mesh equation.

Method 1: Assign a name to the current source voltage. Apply KVL to both of the meshes. Eliminate the current source voltage from the KVL equations.

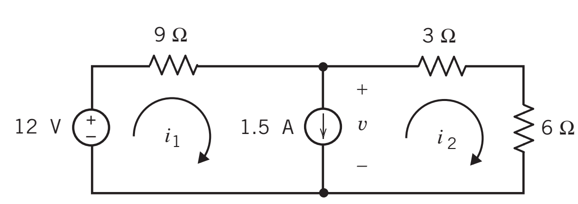

FIGURE 4.6-6
Method 1 of Example 4.6-2.

Figure 4.6-6 shows the circuit after labeling the current source voltage. The KVL equation for mesh 1 is

$$9i_1 + v - 12 = 0$$

The KVL equation for mesh 2 is

$$3i_2 + 6i_2 - v = 0$$

Combining these two equations gives

$$9i_1 + (3i_2 + 6i_2) - 12 = 0 \quad \Rightarrow \quad 9i_1 + 9i_2 = 12$$

Method 2: Apply KVL to the supermesh corresponding to the current source. Shown in Figure 4.6-7, this supermesh is the perimeter of the two meshes that each contain the current source. Apply KVL to the supermesh to get

$$9i_1 + 3i_2 + 6i_2 - 12 = 0 \quad \Rightarrow \quad 9i_1 + 9i_2 = 12$$

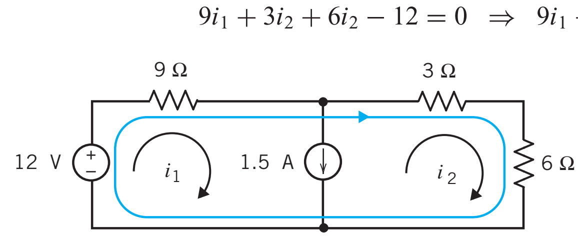

FIGURE 4.6-7
Method 2 of Example 4.6-2.

This is the same equation that was obtained using method 1. Applying KVL to the supermesh is a shortcut for doing three things:

1. Labeling the current source voltage as v

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2. Applying KVL to both meshes that contain the current source
3. Eliminating v from the KVL equations

In summary, the mesh equations are

$$i_1 = i_2 + 1.5$$

and

$$9i_1 + 9i_2 = 12$$

Solving the node equations gives

$$i_1 = 1.4167\text{ A} \quad \text{and} \quad i_2 = -83.3\text{ mA}$$

Exercise 4.6-1 Determine the value of the voltage measured by the voltmeter in Figure E 4.6-1.

Hint: Write and solve a single mesh equation to determine the current in the 3-Ω resistor.

Answer: −4 V

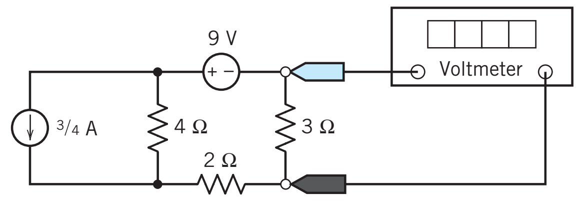

FIGURE E 4.6-1

Exercise 4.6-2 Determine the value of the current measured by the ammeter in Figure E 4.6-2.

Hint: Write and solve a single mesh equation.

Answer: −3.67 A

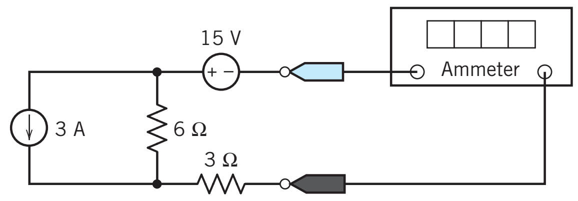

FIGURE E 4.6-2

4.7 Mesh Current Analysis with Dependent Sources

When a circuit contains a dependent source, the controlling current or voltage of that dependent source must be expressed as a function of the mesh currents.

It is then a simple matter to express the controlled current or voltage as a function of the mesh currents. The mesh equations can then be obtained by applying Kirchhoff's voltage law to the meshes of the circuit.

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EXAMPLE 4.7-1 *Mesh Equations and Dependent Sources*

INTERACTIVE EXAMPLE

Consider the circuit shown in Figure 4.7-1*a*. Find the value of the voltage measured by the voltmeter.

Solution

Figure 4.7-1*b* shows the circuit after replacing the voltmeter by an equivalent open circuit and labeling the voltage, v_m, measured by the voltmeter. Figure 4.7-1*c* shows the circuit after numbering the meshes. Let i_1 and i_2 denote the mesh currents in meshes 1 and 2, respectively.

The controlling current of the dependent source, i_a, is the current in a short circuit. This short circuit is common to meshes 1 and 2. The short-circuit current can be expressed in terms of the mesh currents as

$$i_a = i_1 - i_2$$

The dependent source is in only one mesh, mesh 2. The reference direction of the dependent source current does not agree with the reference direction of i_2. Consequently,

$$5i_a = -i_2$$

Solving for i_2 gives

$$i_2 = -5i_a = -5(i_1 - i_2)$$

Therefore

$$-4i_2 = -5i_1 \quad \Rightarrow \quad i_2 = \frac{5}{4}i_1$$

Apply KVL to mesh 1 to get

$$32i_1 - 24 = 0 \quad \Rightarrow \quad i_1 = \frac{3}{4}\text{ A}$$

Consequently, the value of i_2 is

$$i_2 = \frac{5}{4}\left(\frac{3}{4}\right) = \frac{15}{16}\text{ A}$$

Apply KVL to mesh 2 to get

$$32i_2 - v_m = 0 \quad \Rightarrow \quad v_m = 32i_2$$

Finally,

$$v_m = 32\left(\frac{15}{16}\right) = 30\text{ V}$$

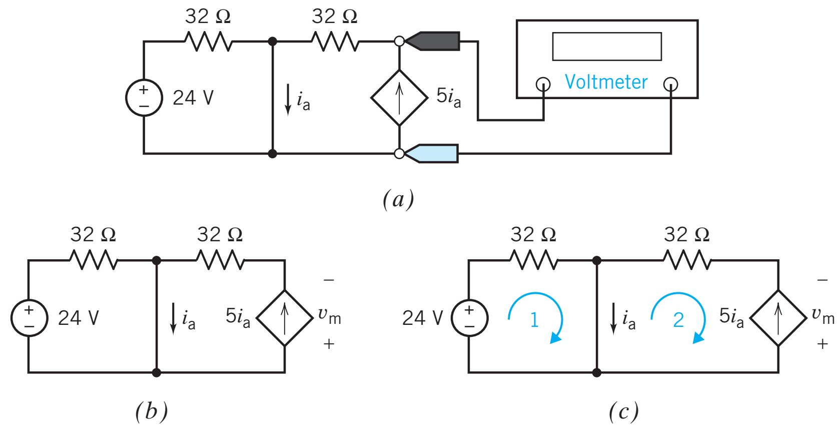

FIGURE 4.7-1 (*a*) The circuit considered in Example 4.7-1. (*b*) The circuit after replacing the voltmeter by an open circuit. (*c*) The circuit after labeling the meshes.

EXAMPLE 4.7-2 *Mesh Equations and Dependent Sources*

INTERACTIVE EXAMPLE

Consider the circuit shown in Figure 4.7-2*a*. Find the value of the gain, *A*, of the CCVS.

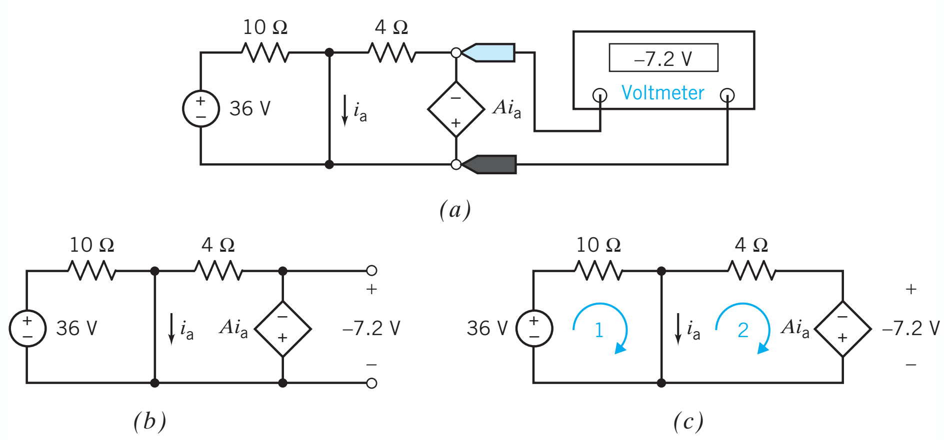

FIGURE 4.7-2 (*a*) The circuit considered in Example 4.7-2. (*b*) The circuit after replacing the voltmeter by an open circuit. (*c*) The circuit after labeling the meshes.

Solution

Figure 4.7-2*b* shows the circuit after replacing the voltmeter by an equivalent open circuit and labeling the voltage measured by the voltmeter. Figure 4.7-2*c* shows the circuit after numbering the meshes. Let i_1 and i_2 denote the mesh currents in meshes 1 and 2, respectively.

The voltage across the dependent source is represented in two ways. It is Ai_a with the + of reference direction at the bottom and −7.2 V with the + at the top. Consequently

$$Ai_a = -(-7.2) = 7.2 \text{ V}$$

The controlling current of the dependent source, i_a, is the current in a short circuit. This short circuit is common to meshes 1 and 2. The short-circuit current can be expressed in terms of the mesh currents as

$$i_a = i_1 - i_2$$

Apply KVL to mesh 1 to get

$$10i_1 - 36 = 0 \quad \Rightarrow \quad i_1 = 3.6 \text{ A}$$

Apply KVL to mesh 2 to get

$$4i_2 + (-7.2) = 0 \quad \Rightarrow \quad i_2 = 1.8 \text{ A}$$

Finally,

$$A = \frac{Ai_a}{i_a} = \frac{Ai_a}{i_1 - i_2} = \frac{7.2}{3.6 - 1.8} = 4 \text{ V/A}$$

4.8 The Node Voltage Method and Mesh Current Method Compared

The analysis of a complex circuit can usually be accomplished by either the node voltage or the mesh current method. The advantage of using these methods is the systematic procedures provided for obtaining the simultaneous equations.

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In some cases one method is clearly preferred over another. For example, when the circuit contains only voltage sources, it is probably easier to use the mesh current method. When the circuit contains only current sources, it will usually be easier to use the node voltage method.

If a circuit has both current sources and voltage sources, it can be analyzed by either method. One approach is to compare the number of equations required for each method. If the circuit has fewer nodes than meshes, it may be wise to select the node voltage method. If the circuit has fewer meshes than nodes, it may be easier to use the mesh current method.

Another point to consider when choosing between the two methods is what information is required. If you need to know several currents, it may be wise to proceed directly with mesh current analysis. Remember, mesh current analysis only works for planar networks.

It is often helpful to determine which method is more appropriate for the problem requirements and to consider both methods.

EXAMPLE 4.8-1 *Mesh Equations*

INTERACTIVE EXAMPLE

Consider the circuit shown in Figure 4.8-1. Find the value of the resistance, R.

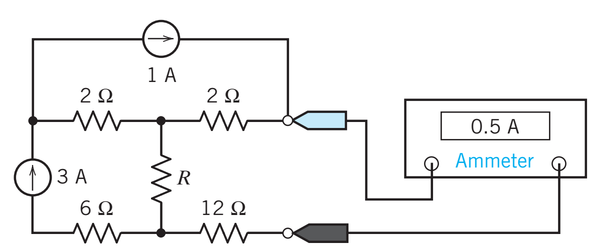

FIGURE 4.8-1 The circuit considered in Example 4.8-1.

Solution

Figure 4.8-2*a* shows the circuit from Figure 4.8-1 after replacing the ammeter by an equivalent short circuit and labeling the current measured by the ammeter. This circuit can be analyzed using mesh equations or using node equations. To decide which will be easier, we first count the nodes and meshes. This circuit has five nodes. Selecting a reference node and then applying KCL at the other four nodes will produce a set of four node equations. The circuit has three meshes. Applying KVL to these three meshes will produce a set of three mesh equations. Hence, analyzing this circuit using mesh equations instead of node equations will produce a smaller set of equations. Further, notice that two of the three mesh currents can be determined directly from the current source currents. This makes the mesh equations easier to solve. We will analyze this circuit by writing and solving mesh equations.

Figure 4.8-2*b* shows the circuit after numbering the meshes. Let i_1, i_2, and i_3 denote the mesh currents in meshes 1, 2, and 3, respectively. The mesh current i_1 is equal to the current in the 1-A current source, so

$$i_1 = 1\text{ A}$$

The mesh current i_2 is equal to the current in the 3-A current source, so

$$i_2 = 3\text{ A}$$

The mesh current i_3 is equal to the current in the short circuit that replaced the ammeter, so

$$i_3 = 0.5\text{ A}$$

Apply KVL to mesh 3 to get

$$2(i_3 - i_1) + 12(i_3) + R(i_3 - i_2) = 0$$

Substituting the values of the mesh currents gives

$$2(0.5 - 1) + 12(0.5) + R(0.5 - 3) = 0 \quad \Rightarrow \quad R = 2\ \Omega$$

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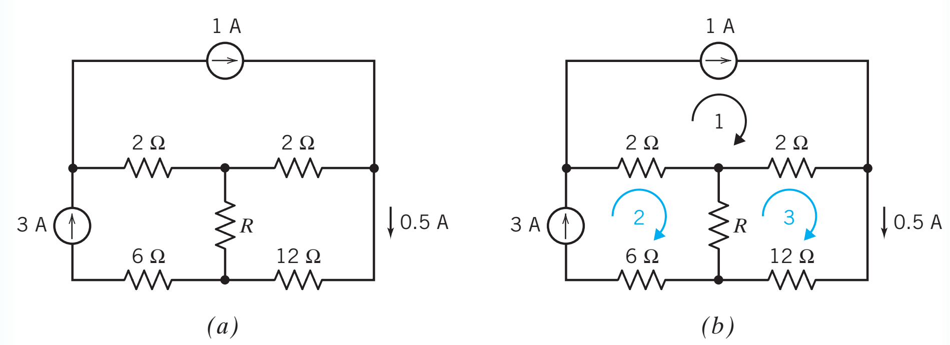

FIGURE 4.8-2 (a) The circuit from Figure 4.8-1 after replacing the ammeter by a short circuit. (*b*) The circuit after labeling the meshes.

EXAMPLE 4.8-2 *Node Equations*

INTERACTIVE EXAMPLE

Consider the circuit shown in Figure 4.8-3. Find the value of the resistance, *R*.

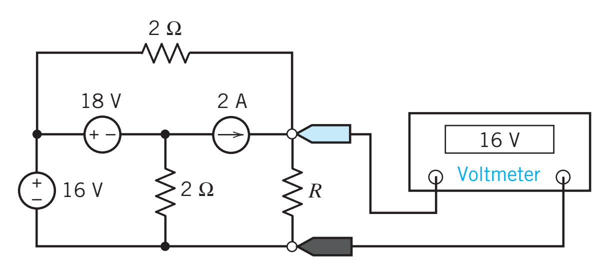

FIGURE 4.8-3 The circuit considered in Example 4.8-2.

Solution

Figure 4.8-4*a* shows the circuit from Figure 4.8-3 after replacing the voltmeter by an equivalent open circuit and labeling the voltage measured by the voltmeter. This circuit can be analyzed using mesh equations or using node equations. To decide which will be easier, we first count the nodes and meshes. This circuit has four nodes. Selecting a reference node and then applying KCL at the other three nodes will produce a set of three node equations. The circuit has three meshes. Applying KVL to these three meshes will produce a set of three mesh equations. Analyzing this circuit using mesh equations requires the same number of equations as are required to analyze the circuit using node equations. Notice that one of the three mesh currents can be determined directly from the current source current, but two of the three node voltages can be determined directly from the voltage source voltages. This makes the node equations easier to solve. We will analyze this circuit by writing and solving node equations.

Figure 4.8-4*b* shows the circuit after selecting a reference node and numbering the other nodes. Let v_1, v_2, and v_3 denote the node voltages at nodes 1, 2, and 3, respectively. The voltage of the 16-V voltage source can be expressed in terms of the node voltages as

$$16 = v_1 - 0 \quad \Rightarrow \quad v_1 = 16 \text{ V}$$

The voltage of the 18-V voltage source can be expressed in terms of the node voltages as

$$18 = v_1 - v_2 \quad \Rightarrow \quad 18 = 16 - v_2 \quad \Rightarrow \quad v_2 = -2 \text{ V}$$

The voltmeter measures the node voltage at node 3, so

$$v_3 = 16 \text{ V}$$

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Apply KCL at node 3 to get

$$\frac{v_1 - v_3}{2} + 2 = \frac{v_3}{R}$$

Substituting the values of the node voltages gives

$$\frac{16 - 16}{2} + 2 = \frac{16}{R} \quad \Rightarrow \quad R = 8\ \Omega$$

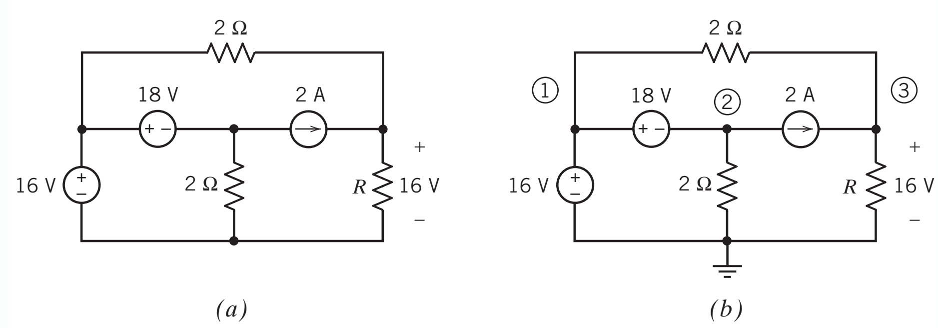


FIGURE 4.8-4 (a) The circuit from Figure 4.8-3 after replacing the voltmeter by an open circuit. (*b*) The circuit after labeling the nodes.

4.9 Mesh Current Analysis Using MATLAB

We have seen that circuits that contain resistors and independent or dependent sources can be analyzed in the following way:

1. Writing a set of node or mesh equations
2. Solving those equations simultaneously

In this section, we will use the computer program MATLAB to solve the equations.

Consider the circuit shown in Figure 4.9-1*a*. This circuit contains a potentiometer. In Figure 4.9-1*b*, the potentiometer has been replaced by a model of a potentiometer. R_p is the resistance of the potentiometer. The parameter a varies from 0 to 1 as the wiper of the potentiometer is moved from one end of the potentiometer to the other. The resistances R_4 and R_5 are described by the equations

$$R_4 = aR_p \tag{4.9-1}$$

and

$$R_5 = (1 - a)R_p \tag{4.9-2}$$

Our objective is to analyze this circuit to determine how the output voltage changes as the position of the potentiometer wiper is changed.

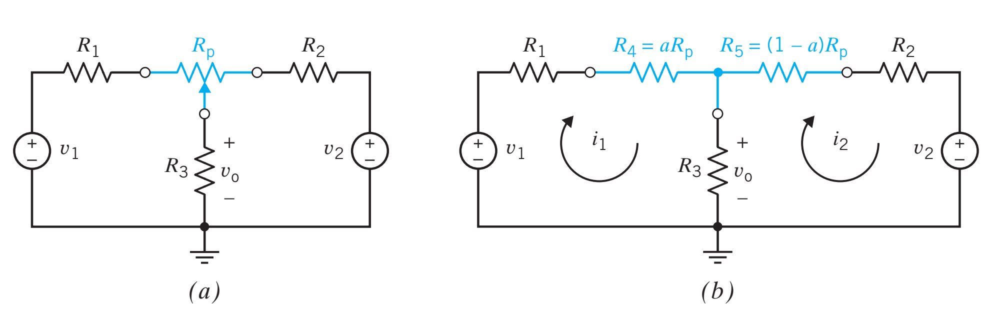


FIGURE 4.9-1 (*a*) A circuit that contains a potentiometer and (*b*) an equivalent circuit formed by replacing the potentiometer by a model of a potentiometer ($0 \le a \le 1$).

The circuit in Figure 4.9-1b can be represented by mesh equations as

$$\begin{aligned} R_1 i_1 + R_4 i_1 + R_3(i_1 - i_2) - v_1 &= 0 \\ R_5 i_2 + R_2 i_2 + [v_2 - R_3(i_1 - i_2)] &= 0 \end{aligned} \tag{4.9-3}$$

These mesh equations can be rearranged as

$$\begin{aligned} (R_1 + R_4 + R_3)i_1 - R_3 i_2 &= v_1 \\ -R_3 i_1 + (R_5 + R_2 + R_3)i_2 &= -v_2 \end{aligned} \tag{4.9-4}$$

Substituting Eqs. 4.9-1 and 4.9-2 into Eq. 4.9-4 gives

$$\begin{aligned} (R_1 + aR_p + R_3)i_1 - R_3 i_2 &= v_1 \\ -R_3 i_1 + [(1-a)R_p + R_2 + R_3]i_2 &= -v_2 \end{aligned} \tag{4.9-5}$$

```
% mesh.m solves mesh equations
%--------------------------------------------------------------
%  Enter values of the parameters that describe the circuit.
%--------------------------------------------------------------
                       % circuit parameters
R1=1000;               % ohms
R2=1000;               % ohms
R3=5000;               % ohms
V1= 15;                % volts
V2=-15;                % volts

                       % potentiometer parameters
Rp=20e3;               % ohms
%--------------------------------------------------------------
%   the parameter a varies from 0 to 1 in 0.05 increments.
%--------------------------------------------------------------
a=0:0.05:1;            % dimensionless

for k=1:length(a)
    %----------------------------------------------------------
    % Here is the mesh equation, RI=V:
    %----------------------------------------------------------
    R = [R1+a(k)*Rp+R3       -R3;                    % -------
               -R3       (1-a(k))*Rp+R2+R3];         % eqn.
    V=[ V1;                                          % 4.9-6
         -V2];                                       % -------
    %----------------------------------------------------------
    % Tell MATLAB to solve the mesh equation:
    %----------------------------------------------------------
    I=V /R;
    %----------------------------------------------------------
    % Calculate the output voltage from the mesh currents.
    %----------------------------------------------------------

    Vo(k)=R3*(I(1)-I(2)); % eqn. 4.9-7

end
%--------------------------------------------------------------
%   Plot Vo versus a
%--------------------------------------------------------------
plot(a, Vo)
axis([0 1 -15 15])
xlabel( a, dimensionless )
ylabel( Vo, V )
```

FIGURE 4.9-2 MATLAB input file used to analyze the circuit shown in Figure 4.9-1.

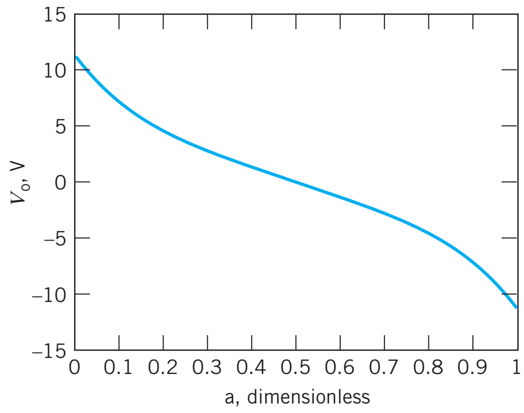


FIGURE 4.9-3
Plot of v_o versus a for the circuit shown in Figure 4.9-1.

Equation 4.9-5 can be written using matrices as

$$\begin{bmatrix} R_1 + aR_P + R_3 & -R_3 \\ -R_3 & (1-a)R_P + R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} v_1 \\ -v_2 \end{bmatrix} \qquad (4.9\text{-}6)$$

Next, i_1 and i_2 are calculated by using MATLAB to solve the mesh equation, Eq. 4.9-6. Then the output voltage is calculated as

$$v_o = R_3(i_1 - i_2) \qquad (4.9\text{-}7)$$

Figure 4.9-2 shows the MATLAB input file. The parameter a varies from 0 to 1 in increments of 0.05. At each value of a, MATLAB solves Eq. 4.9-6 and then uses Eq. 4.9-7 to calculate the output voltage. Finally, MATLAB produces the plot of v_o versus a that is shown in Figure 4.9-3.

4.10 How Can We Check . . . ?

Engineers are frequently called upon to check that a solution to a problem is indeed correct. For example, proposed solutions to design problems must be checked to confirm that all of the specifications have been satisfied. In addition, computer output must be reviewed to guard against data-entry errors, and claims made by vendors must be examined critically.

Engineering students are also asked to check the correctness of their work. For example, occasionally just a little time remains at the end of an exam. It is useful to be able to quickly identify those solutions that need more work.

The following examples illustrate techniques useful for checking the solutions of the sort of problem discussed in this chapter.

EXAMPLE 4.10-1 *How Can We Check Node Voltages?*

The circuit shown in Figure 4.10-1*a* was analyzed using PSpice. The PSpice output file, Figure 4.10-1*b*, includes the node voltages of the circuit. **How can we check** that these node voltages are correct?

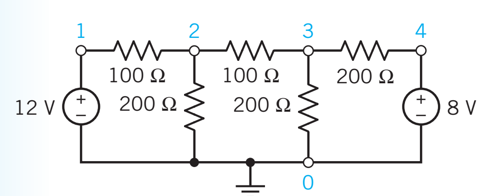
Solution

The node equation corresponding to node 2 is

$$\frac{V(2) - V(1)}{100} + \frac{V(2)}{200} + \frac{V(2) - V(3)}{100} = 0$$

where, for example, $V(2)$ is the node voltage at node 2. When the node voltages from Figure 4.10-1*b* are substituted into the left-hand side of this equation, the result is

$$\frac{7.2727 - 12}{100} + \frac{7.2727}{200} + \frac{7.2727 - 5.0909}{100} = 0.011$$



(a)

```
Node Voltage Example

V1    1    0    12
R1    1    2    100
R2    2    0    200
R3    2    3    200
R4    3    0    200
R5    3    4    200
V2    4    0    8

.END

 NODE VOLTAGES

 NODE VOLTAGE

 (1)    12.0000
 (2)     7.2727
 (3)     5.0909
 (4)     8.0000
```

(b)

FIGURE 4.10-1 (*a*) A circuit and (*b*) the node voltages calculated using PSpice. The bottom node has been chosen as the reference node, which is indicated by the ground symbol and the node number 0. The voltages and resistors have units of voltages and ohms, respectively.

The right-hand side of this equation should be 0 instead of 0.011. It looks like something is wrong. Is a current of only 0.011 negligible? Probably not in this case. If the node voltages were correct, then the currents of the 100-Ω resistors would be 0.047 A and 0.022 A, respectively. The current of 0.011 A does not seem negligible when compared to currents of 0.047 A and 0.022 A.

Is it possible that PSpice would calculate the node voltages incorrectly? Probably not, but the PSpice input file could easily contain errors. In this case, the value of the resistance connected between nodes 2 and 3 has been mistakenly specified to be 200 Ω. After changing this resistance to 100 Ω, PSpice calculates the node voltages to be

$$V(1) = 12.0,\ \ V(2) = 7.0,\ \ V(3) = 5.5,\ \ V(4) = 8.0$$

Substituting these voltages into the node equation gives

$$\frac{7.0 - 12.0}{100} + \frac{7.0}{200} + \frac{7.0 - 5.5}{100} = 0.0$$

so these node voltages do satisfy the node equation corresponding to node 2.

EXAMPLE 4.10-2 *How Can We Check Mesh Currents?*

The circuit shown in Figure 4.10-2*a* was analyzed using PSpice. The PSpice output file, Figure 4.10-2*b*, includes the mesh currents of the circuit. **How can we check** that these mesh currents correct?

(The PSpice output file will include the currents through the voltage sources. Recall that PSpice uses the passive convention, so the current in the 8-V source will be $-i_1$ instead of i_1. The two 0-V sources have been added to include mesh currents i_2 and i_3 in the PSpice output file.)

Solution

The mesh equation corresponding to mesh 2 is

$$200(i_2 - i_1) + 500i_2 + 250(i_2 - i_3) = 0$$

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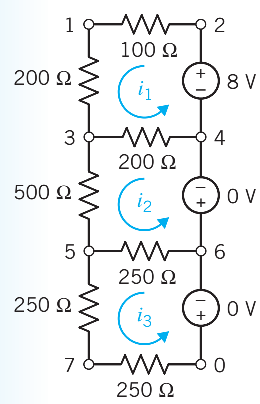

(a)

```
Mesh Current Example

R1    1    2    100
R2    1    3    200
V1    2    4    8
R3    3    4    200
R5    3    5    500
V2    4    6    0
R6    5    6    250
R7    5    7    250
V3    6    0    0
R8    7    0    250

.END

 MESH CURRENTS

 NAME    CURRENT

 I1     1.763E-02
 I2    -4.068E-03
 I3    -1.356E-03
```

(b)

FIGURE 4.10-2 (*a*) A circuit and (*b*) the mesh currents calculated using PSpice. The voltages and resistances are given in volts and ohms, respectively.

When the mesh currents from Figure 4.10-2*b* are substituted into the left-hand side of this equation, the result is

$$200(-0.004068 - 0.01763) + 500(-0.004068) + 250(-0.004068 - (-0.001356)) = 1.629$$

The right-hand side of this equation should be 0 instead of 1.629. It looks like something is wrong. Most likely, the PSpice input file contains an error. This is indeed the case. The nodes of both 0-V voltage sources have been entered in the wrong order. Recall that the first node should be the positive node of the voltage source. After correcting this error, PSpice gives

$$i_1 = 0.01763, \quad \mathrm{i}_2 = 0.004068, \quad i_3 = 0.001356$$

Using these values in the mesh equation gives

$$200(0.004068 - 0.01763) + 500(0.004068) + 250(0.004068 - 0.001356) = 0.0$$

These mesh currents do indeed satisfy the mesh equation corresponding to mesh 2.

4.11 Design Example

Potentiometer Angle Display

A circuit is needed to measure and display the angular position of a potentiometer shaft. The angular position, θ, will vary from $-180°$ to $180°$.

Figure 4.11-1 illustrates a circuit that could do the job. The +15-V and −15-V power supplies, the potentiometer, and resistors R_1 and R_2 are used to obtain a voltage, v_i, that is proportional to θ. The amplifier is used to change the constant of proportionality to obtain

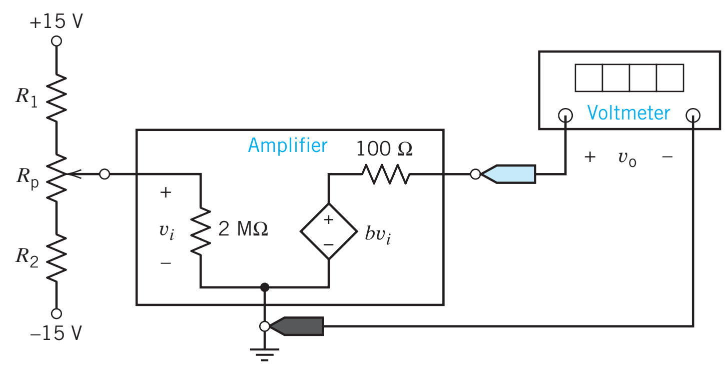

FIGURE 4.11-1 Proposed circuit for measuring and displaying the angular position of the potentiometer shaft.

a simple relationship between θ and the voltage, v_o, displayed by the voltmeter. In this example, the amplifier will be used to obtain the relationship

$$v_o = k \cdot \theta \quad \text{where} \quad k = 0.1\frac{\text{volt}}{\text{degree}} \tag{4.11-1}$$

so that θ can be determined by multiplying the meter reading by 10. For example, a meter reading of -7.32 V indicates that $\theta = -73.2^\circ$.

Describe the Situation and the Assumptions

The circuit diagram in Figure 4.11-2 is obtained by modeling the power supplies as ideal voltage sources, the voltmeter as an open circuit, and the potentiometer by two resistors. The parameter, a, in the model of the potentiometer varies from 0 to 1 as θ varies from -180° to 180°. That means

$$a = \frac{\theta}{360^\circ} + \frac{1}{2} \tag{4.11-2}$$

Solving for θ gives

$$\theta = \left(a - \frac{1}{2}\right) \cdot 360^\circ \tag{4.11-3}$$

State the Goal

Specify values of resistors R_1 and R_2, the potentiometer resistance R_p, and the amplifier gain b that will cause the meter voltage, v_o, to be related to the angle θ by Eq. 4.11-1.

Generate a Plan

Analyze the circuit shown in Figure 4.11-2 to determine the relationship between v_i and θ. Select values of R_1, R_2, and R_p. Use these values to simplify the relationship between between v_i and θ. If possible, calculate the value of b that will cause the meter voltage, v_o, to be related to the angle θ by Eq. 4.11-1. If this isn't possible, adjust the values of R_1, R_2, and R_p and try again.

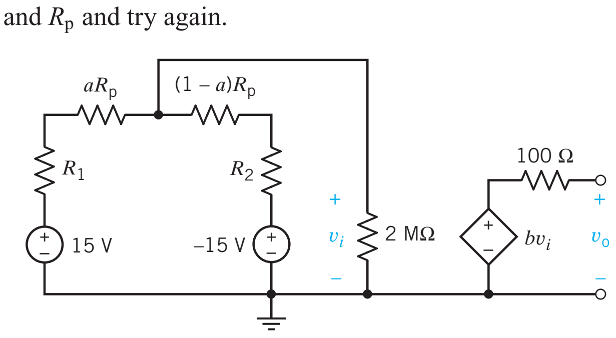

FIGURE 4.11-2 Circuit diagram containing models of the power supplies, voltmeter, and potentiometer.

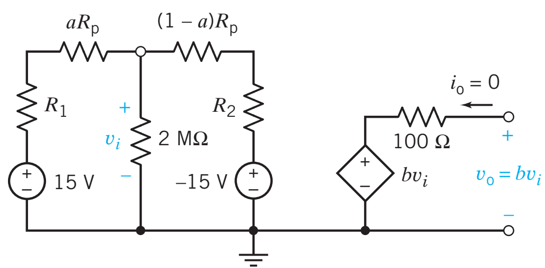

FIGURE 4.11-3
The redrawn circuit showing the mode v_i.

Act on the Plan

The circuit has been redrawn in Figure 4.11-3. A single node equation will provide the relationship between between v_i and θ:

$$\frac{v_i}{2\ \text{M}\Omega} + \frac{v_i - 15}{R_1 + aR_p} + \frac{v_i - (-15)}{R_2 + (1-a)R_p} = 0$$

Solving for v_i gives

$$v_i = \frac{2\ \text{M}\Omega(R_p(2a-1) + R_1 - R_2)15}{(R_1 + aR_p)(R_2 + (1-a)R_p) + 2\ \text{M}\Omega(R_1 + R_2 + R_p)} \tag{4.11-4}$$

This equation is quite complicated. Let's put some restrictions on R_1, R_2, and R_p that will make it possible to simplify this equation. First, let $R_1 = R_2 = R$. Second, require that both R and R_p be much smaller than 2 MΩ (for example, $R < 20$ kΩ). Then

$$(R + aR_p)(R + (1-a)R_p) \ll 2\ \text{M}\Omega(2R + R_p)$$

That is, the first term in the denominator of the left side of Eq. 4.11-4 is negligible compared to the second term. Equation 4.11-4 can be simplified to

$$v_i = \frac{R_p(2a-1)15}{2R + R_p}$$

Next, using Eq. 4.11-3,

$$v_i = \left(\frac{R_p}{2R + R_p}\right)\left(\frac{15\ \text{V}}{180^\circ}\right)\theta$$

It is time to pick values for R and R_p. Let $R = 5$ kΩ and $R_p = 10$ kΩ; then

$$v_i = \left(\frac{7.5\ \text{V}}{180^\circ}\right)\theta$$

Referring to Figure 4.11-2, the amplifier output is given by

$$v_o = bv_i \tag{4.11-5}$$

so

$$v_o = b\left(\frac{7.5\ \text{V}}{180^\circ}\right)\theta$$

Comparing this equation to Eq. 4.11-1 gives

$$b\left(\frac{7.5\ \text{V}}{180^\circ}\right) = 0.1\frac{\text{volt}}{\text{degree}}$$

or

$$b = \frac{180}{7.5}(0.1) = 2.4$$

The final circuit is shown in Figure 4.11-4.

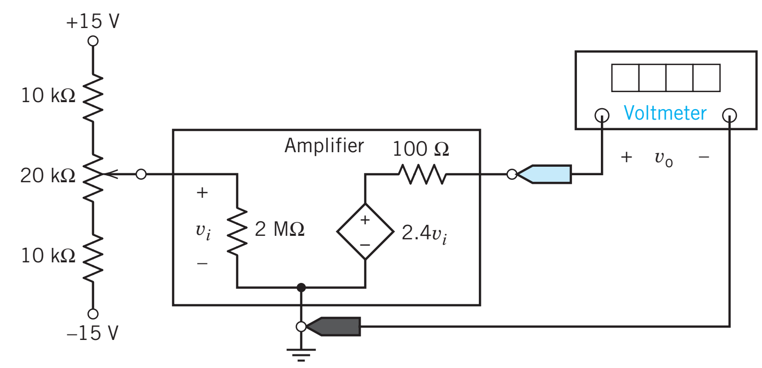

FIGURE 4.11-4 The final designed circuit.

Verify the Proposed Solution

As a check, suppose $\theta = 150^\circ$. From Eq. 4.11-2, we see that

$$a = \frac{150^\circ}{360^\circ} + \frac{1}{2} = 0.9167$$

Using Eq. 4.11-4, we calculate

$$v_i = \frac{2\ \text{M}\Omega(10\ \text{k}\Omega(2 \times 0.9167 - 1))15}{(5\ \text{k}\Omega + 0.9167 \times 10\ \text{k}\Omega)(5\ \text{k}\Omega + (1 - 0.9167)10\ \text{k}\Omega) + 2\ \text{M}\Omega(2 \times 5\ \text{k}\Omega + 10\ \text{k}\Omega)}$$
$$= 6.24$$

Finally, Eq. 4.11-5 indicates that the meter voltage will be

$$v_o \times 2.4 \cdot 6.24 = 14.98$$

This voltage will be interpreted to mean that the angle was

$$\theta = 10 \cdot v_o = 149.8^\circ$$

which is correct to three significant digits.

4.12 Summary

- The node voltage method of circuit analysis identifies the nodes of a circuit where two or more elements are connected. When the circuit consists of only resistors and current sources, the following procedure is used to obtain the node equations.
  1. We choose one node as to the reference node. Label the node voltages at the other nodes.
  2. Express element currents as functions of the node voltages. Figure 4.12-1*a* illustrates the relationship between the current in a resistor and the voltages at the nodes of the resistor.
  3. Apply KCL at all nodes except for the reference node. Solution of the simultaneous equations results in knowledge of the node voltages. All the voltages and currents in the circuit can be determined once the node voltages are known.
- When a circuit has voltage sources as well as current sources, we can still use the node voltage method by utilizing the concept of a supernode. A supernode is a "large node" that includes two nodes connected by a known voltage source. If the voltage source is directly connected between a node q and the reference node, we may set $v_q = v_s$ and write the KCL equations at the remaining nodes.
- If the circuit contains a dependent source, we first express the controlling voltage or current of the dependent source as a function of the node voltages. Next, we express the controlled voltage or current as a function of the node voltages. Finally, we apply KCL to nodes and supernodes.
- Mesh current analysis is accomplished by applying KVL to the meshes of a planar circuit. When the circuit consists of only resistors and voltage sources, the following procedure is used to obtain the mesh equations.
  1. Label the mesh currents.
  2. Express element voltages as functions of the mesh currents. Figure 4.12-1*b* illustrates the relationship between the voltage across a resistor and the currents of the meshes that include the resistor.

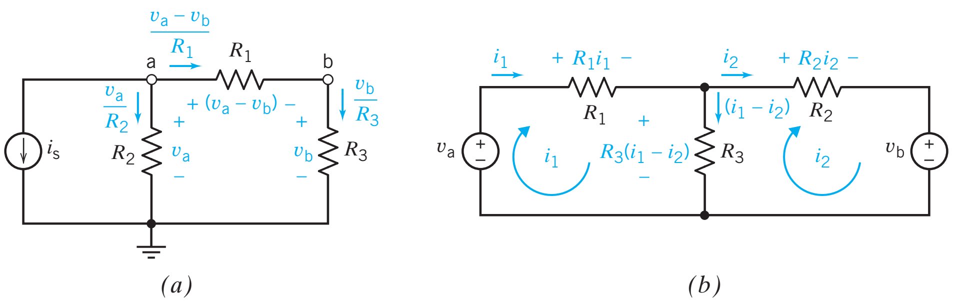

FIGURE 4.12-1 Expressing resistor currents and voltages in terms of (*a*) node voltage or (*b*) mesh currents.

3. Apply KVL to all meshes.
   Solution of the simultaneous equations results in knowledge of the mesh currents. All the voltages and currents in the circuit can be determined once the mesh currents are known.

- If a current source is common to two adjoining meshes, we define the interior of the two meshes as a supermesh. We then write the mesh current equation around the periphery of the supermesh. If a current source appears at the periphery of only one mesh, we may define that mesh current as equal to the current of the source, accounting for the direction of the current source.
- If the circuit contains a dependent source, we first express the controlling voltage or current of the dependent source as a function of the mesh currents. Next, we express the controlled voltage or current as a function of the mesh currents. Finally, we apply KVL to meshes and supermeshes.
- In general, either node voltage or mesh current analysis can be used to obtain the currents or voltages in a circuit. However, a circuit with fewer node equations than mesh current equations may require that we select the node voltage method. Conversely, mesh current analysis is readily applicable for a circuit with fewer mesh current equations than node voltage equations.
- MATLAB greatly reduces the drudgery of solving node or mesh equations.

PROBLEMS

Section 4.2 Node Voltage Analysis of Circuits with Current Sources

P 4.2-1 The node voltages in the circuit of Figure P 4.2-1 are $v_1 = -4$ V and $v_2 = 2$ V. Determine i, the current of the current source.
Answer: $i = 1.5$ A

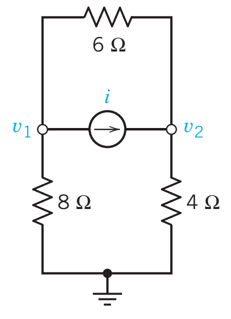

FIGURE P 4.2-1

P 4.2-2 Determine the node voltages for the circuit of Figure P 4.2-2.
Answer: $v_1 = 2$ V, $v_2 = 30$ V, and $v_3 = 24$ V

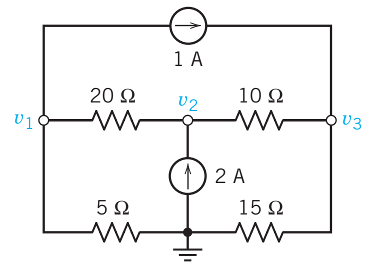

FIGURE P 4.2-2

P 4.2-3 The node voltages in the circuit of Figure P 4.2-3 are $v_1 = 4$ V, $v_2 = 15$ V, and $v_3 = 18$ V. Determine i_1 and i_2, the currents of the current sources.
Answer: $i_1 = -2$ A and $i_2 = 2$ A

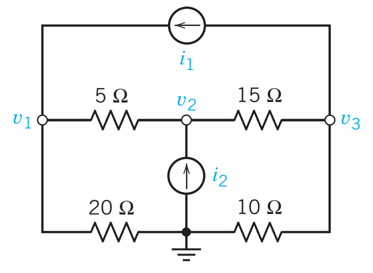

FIGURE P 4.2-3

P 4.2-4 Consider the circuit shown in Figure P 4.2-4. Find values of the resistances R_1 and R_2 that cause the voltages v_1 and v_2 to be $v_1 = 1$ V and $v_2 = 2$ V.

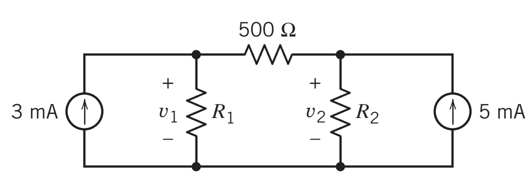

FIGURE P 4.2-4

P 4.2-5 Find the voltage v for the circuit shown in Figure P 4.2-5.
Answer: $v = 21.7$ mV

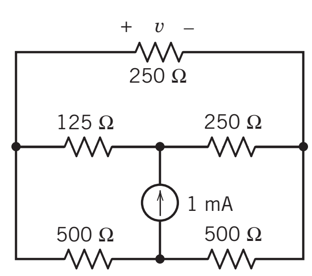

FIGURE P 4.2-5

P4.2-6 Simplify the circuit shown in Figure P 4.2-6 by replacing series and parallel resistors with equivalent resistors; then analyze the simplified circuit by writing and solving node equations. (a) Determine the power supplied by each current source. (b) Determine the power received by the 12-Ω resistor.

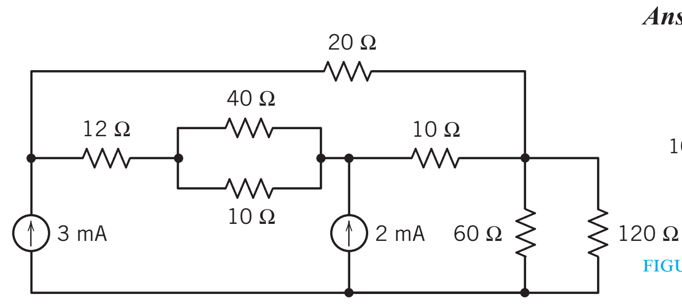

FIGURE P 4.2-6

P4.2-7 The node voltages in the circuit shown in Figure P 4.2-7 are $v_a = 7$ V and $v_b = 10$ V. Determine values of the current source current, i_s, and the resistance, R.

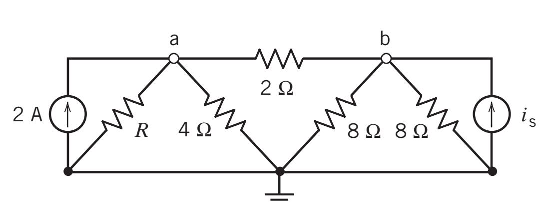

FIGURE P 4.2-7

Section 4.3 Node Voltage Analysis of Circuits with Current and Voltage Sources

P4.3-1 The voltmeter in Figure P 4.3-1 measures v_c, the node voltage at node c. Determine the value of v_c.
Answer: $v_c = 2$ V

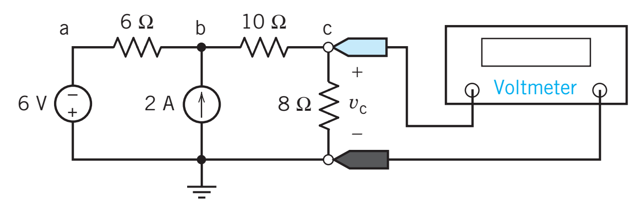

FIGURE P 4.3-1

P4.3-2 The voltages v_a, v_b, v_c, and v_d in Figure P 4.3-2 are the node voltages corresponding to nodes a, b, c, and d. The current i is the current in a short circuit connected between nodes b and c. Determine the values of v_a, v_b, v_c, and v_d and of i.
Answer: $v_a = -12$ V, $v_b = v_c = 4$ V, $v_d = -4$ V, $i = 2$ mA

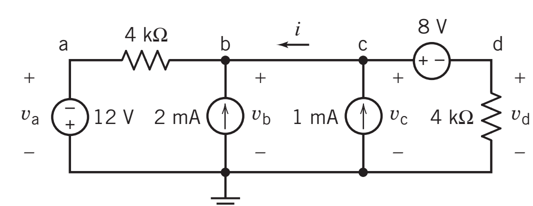

FIGURE P 4.3-2

P4.3-3 Determine the node voltage v_a for the circuit of Figure P 4.3-3.
Answer: $v_a = 7$ V

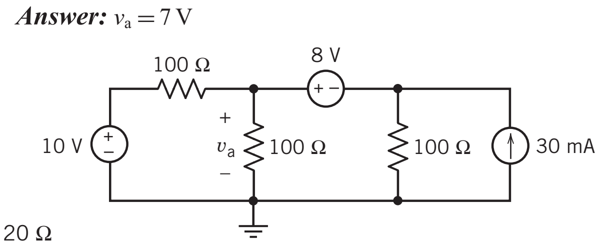

FIGURE P 4.3-3

P4.3-4 Determine the node voltage v_a for the circuit of Figure P 4.3-4.
Answer: $v_a = 4$ V

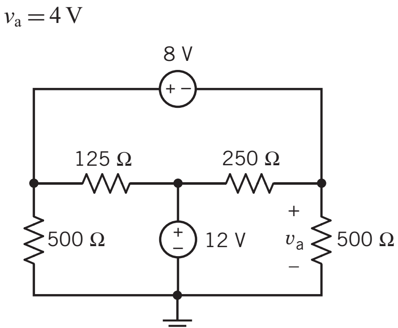

FIGURE P 4.3-4

P1: IML/OVY P2: IML/OVY QC: IML/OVY T1: IML
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P 4.3-5 The voltages v_a, v_b, and v_c in Figure P 4.3-5 are the node voltages corresponding to nodes a, b, and c. The values of these voltages are:

$$v_a = 12\ \text{V},\ v_b = 9.882\ \text{V},\ \text{and}\ v_c = 5.294\ \text{V}$$

Determine the power supplied by the voltage source.

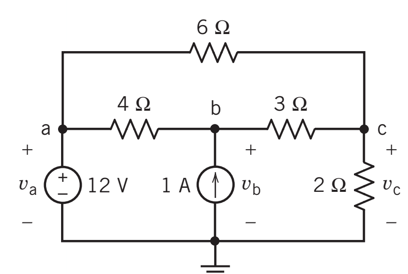

FIGURE P 4.3-5

P 4.3-6 The voltmeter in the circuit of Figure P 4.3-6 measures a node voltage. The value of that node voltage depends on the value of the resistance R.

(a) Determine the value of the resistance R that will cause the voltage measured by the voltmeter to be 4 V.

(b) Determine the voltage measured by the voltmeter when $R = 1.2\ \text{k}\Omega = 1200\ \Omega$.

Answers: (a) 6 kΩ (b) 2 V

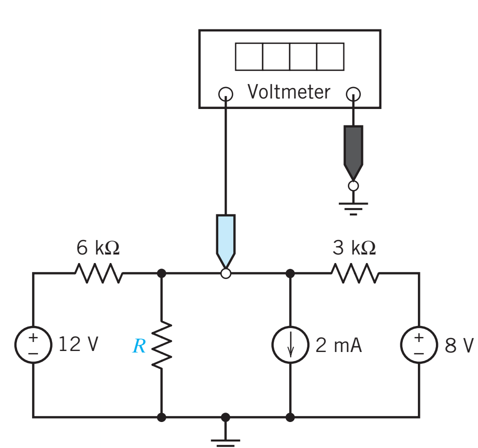

FIGURE P 4.3-6

P 4.3-7 Determine the values of the node voltages, v_1 and v_2, in Figure P 4.3-7. Determine the values of the currents i_a and i_b.

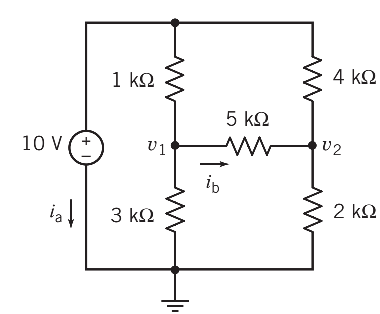

FIGURE P 4.3-7

P 4.3-8 The circuit shown in Figure P 4.3-8 has two inputs, v_1 and v_2, and one output, v_o. The output is related to the input by the equation

$$v_o = av_1 + bv_2$$

where a and b are constants that depend on R_1, R_2 and R_3.

(a) Determine the values of the coefficients a and b when $R_1 = 10\ \Omega$, $R_2 = 40\ \Omega$ and $R_3 = 8\ \Omega$.

(b) Determine the values of the coefficients a and b when $R_1 = R_2$ and $R_3 = R_1 \parallel R_2$.

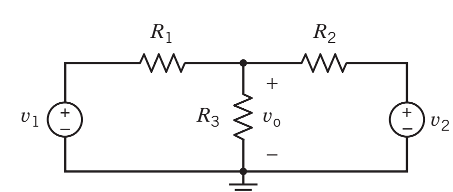

FIGURE P 4.3-8

P 4.3-9 Determine the values of the node voltages of the circuit shown in Figure P 4.3-9.

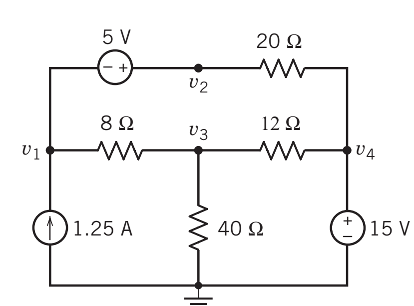

FIGURE P 4.3-9

P 4.3-10 Figure P 4.3-10 shows a measurement made in the laboratory. Your lab partner forgot to record the values of R_1, R_2, and R_3. He thinks that the two resistors were 10-kΩ resistors and the other was a 5-kΩ resistor. Is this possible? Which resistor is the 5-kΩ resistor?

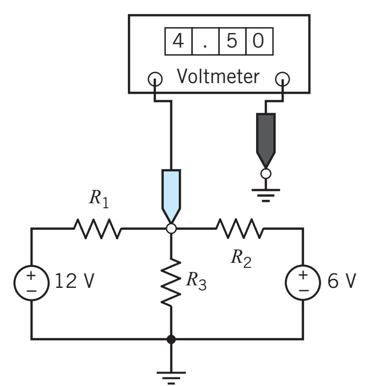

FIGURE P 4.3-10

***P 4.3-11** Determine the values of the node voltages of the circuit shown in Figure P 4.3-11.

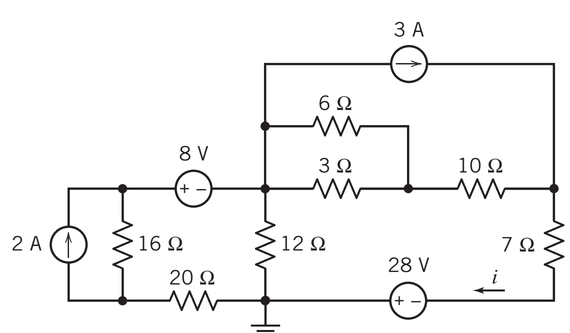

FIGURE P 4.3-11

P 4.3-12 Determine the values of the node voltages of the circuit shown in Figure P 4.3-12.

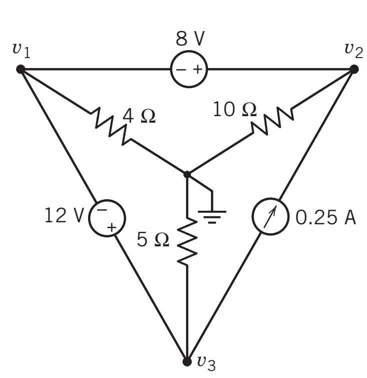

FIGURE P 4.3-12

Section 4.4 Node Voltage Analysis with Dependent Sources

P 4.4-1 The voltages v_a, v_b, and v_c in Figure P 4.4-1 are the node voltages corresponding to nodes a, b, and c. The values of these voltages are:

$$v_a = 8.667 \text{ V}, \ v_b = 2 \text{ V}, \text{ and } v_c = 10 \text{ V}$$

Determine the value of A, the gain of the dependent source.

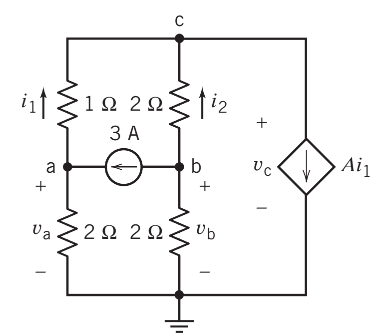

FIGURE P 4.4-1

P 4.4-2 Find i_b for the circuit shown in Figure P 4.4-2.
Answer: $i_b = -12$ mA

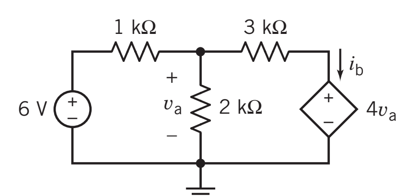

FIGURE P 4.4-2

P 4.4-3 Determine the node voltage v_b for the circuit of Figure P 4.4-3.
Answer: $v_b = 1.5$ V

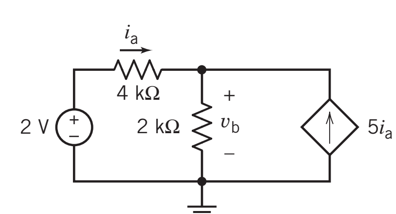

FIGURE P 4.4-3

P 4.4-4 The circled numbers in Figure P 4.4-4 are node numbers. The node voltages of this circuit are $v_1 = 10$ V, $v_2 = 14$ V, and $v_3 = 12$ V.

(a) Determine the value of the current i_b.

(b) Determine the value of r, the gain of the CCVS.

Answers: (a) −2 A (b) 4 V/A

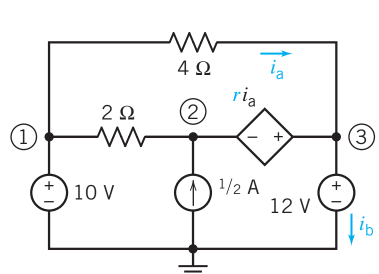

FIGURE P 4.4-4

P 4.4-5 Determine the value of the current i_x in the circuit of Figure P 4.4-5.

Answer: $i_x = 2.4$ A

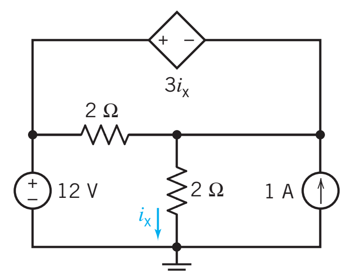

FIGURE P 4.4-5

P 4.4-6 Determine the power supplied by the 12-V voltage source in Figure P 4.4-6.

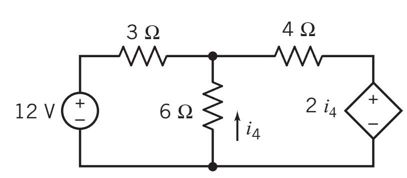

FIGURE P 4.4-6

P 4.4-7 Determine the value of the current i_c in Figure P 4.4-7.

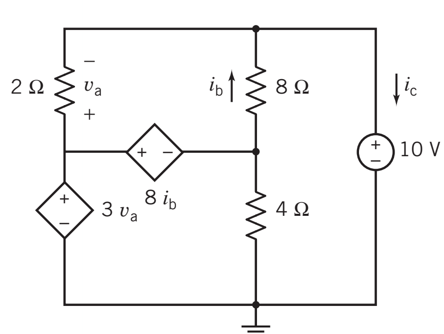

FIGURE P 4.4-7

P 4.4-8 Determine the value of the power supplied by the dependent source in Figure P 4.4-8.

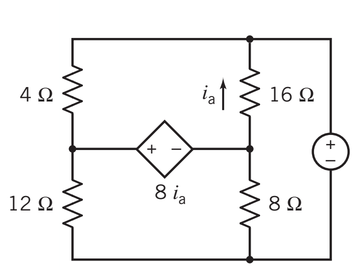

FIGURE P 4.4-8

P 4.4-9 The node voltages in the circuit shown in Figure P 4.4-9 are

$$v_1 = 4 \text{ V}, \; v_2 = 0 \text{ V}, \text{ and } v_3 = -6 \text{ V}$$

Determine the values of the resistance, R, and of the gain, b, of the CCCS.

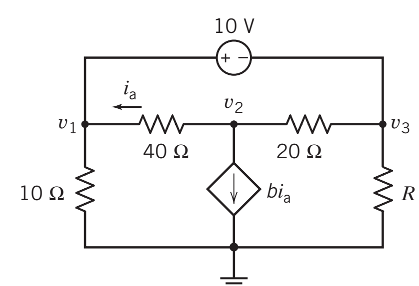

FIGURE P 4.4-9

P 4.4-10 The value of the node voltage at node b in the circuit shown in Figure P 4.4-10 is $v_b = 18$ V.

(a) Determine the value of A, the gain of the dependent source.

(b) Determine the power supplied by the dependent source.

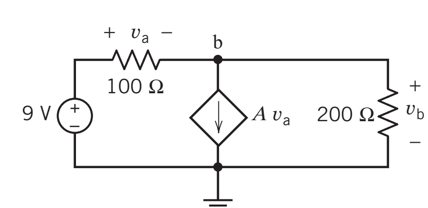

FIGURE P 4.4-10

***P 4.4-11** Determine the power supplied by the dependent source in the circuit shown in Figure P 4.4-11.

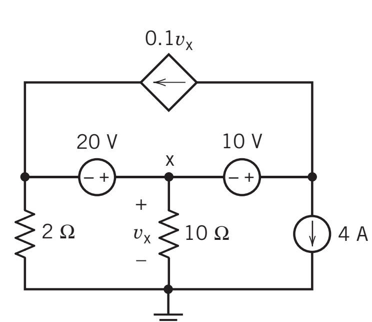

FIGURE P 4.4-11

***P 4.4-12** Determine values of the node voltages, v_1, v_2, v_3, v_4, and v_5, in the circuit shown in Figure P 4.4-12.

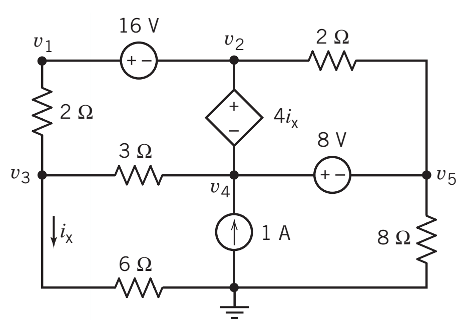

FIGURE P 4.4-12

***P 4.4-13** Determine values of the node voltages, v_1, v_2, v_3, v_4, and v_5, in the circuit shown in Figure P 4.4-13.

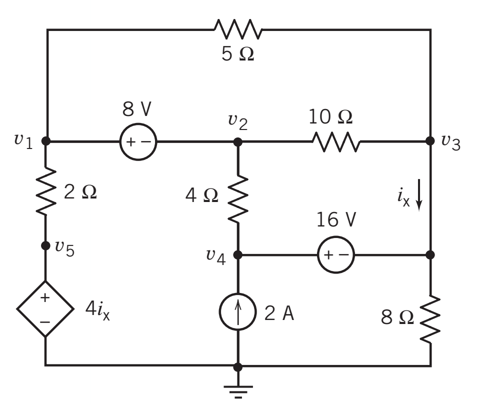

FIGURE P 4.4-13

***P 4.4-14** Determine values of the node voltages, v_1, v_2, v_3, and v_4, in the circuit shown in Figure P 4.4-14.

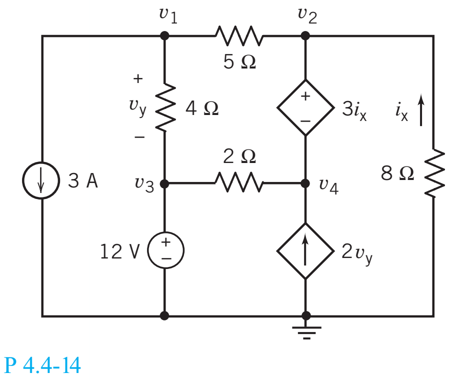

FIGURE P 4.4-14

P 4.4-15 The voltages v_1, v_2, v_3, and v_4 are the node voltages corresponding to nodes 1, 2, 3, and 4 in Figure P 4.4-15. Determine the values of these node voltages.

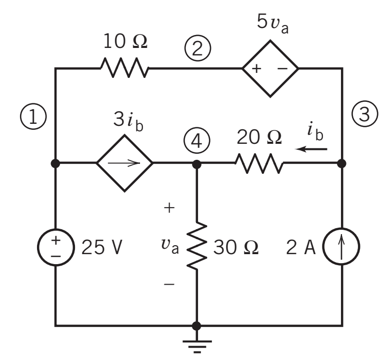

FIGURE P 4.4-15

P 4.4-16 The voltages v_1, v_2, v_3, and v_4 in Figure P 4.4-16 are the node voltages corresponding to nodes 1, 2, 3, and 4. The values of these voltages are

$$v_1 = 10\text{ V},\ v_2 = 75\text{ V},\ v_3 = -15\text{ V},\text{ and } v_4 = 22.5\text{ V}$$

Determine the values of the gains of the dependent sources, A and B, and of the resistance R_1.

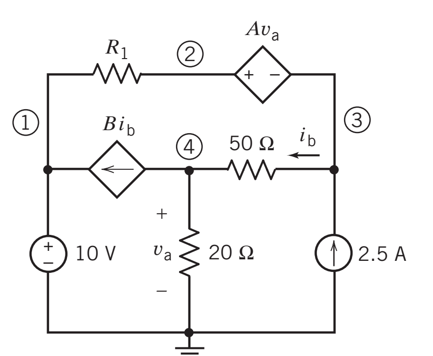

FIGURE P 4.4-16

P 4.4-17 The voltages v_1, v_2, and v_3 in Figure P 4.4-17 are the node voltages corresponding to nodes 1, 2, and 3. The values of these voltages are

$$v_1 = 12\text{ V},\ v_2 = 21\text{ V},\text{ and } v_3 = -3\text{ V}$$

(a) Determine the values of the resistances R_1 and R_2.
(b) Determine the power supplied by each source.

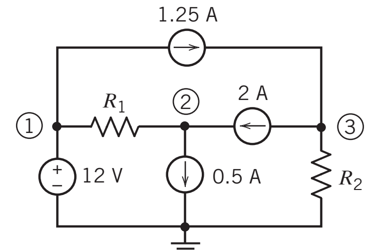

FIGURE P 4.4-17

P 4.4-18 The voltages v_1, v_2, and v_3 in Figure P 4.4-18 are the node voltages corresponding to nodes 1, 2, and 3. The values of these voltages are

$$v_1 = 12\text{ V}, v_2 = 9.6\text{ V}, \text{and } v_3 = -1.33\text{ V}$$

(a) Determine the values of the resistances R_1 and R_2.

(b) Determine the power supplied by each source.

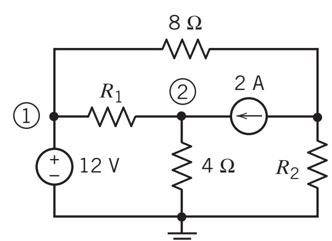

FIGURE P 4.4-18

Section 4.5 Mesh Current Analysis with Independent Voltage Sources

P 4.5-1 Determine the mesh currents, i_1, i_2, and i_3, for the circuit shown in Figure P 4.5-1.
Answers: $i_1 = 3$ A, $i_2 = 2$ A, and $i_3 = 4$ A

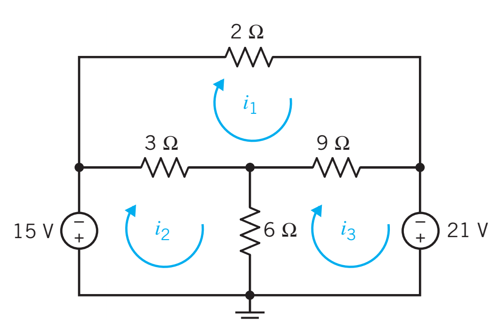

FIGURE P 4.5-1

P 4.5-2 The values of the mesh currents in the circuit shown in Figure P 4.5-2 are $i_1 = 2$ A, $i_2 = 3$ A, and $i_3 = 4$ A. Determine the values of the resistance R and of the voltages v_1 and v_2 of the voltage sources.
Answers: $R = 12\ \Omega$, $v_1 = -4$ V, and $v_2 = -28$ V

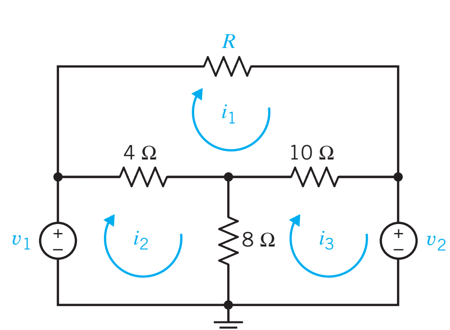

FIGURE P 4.5-2

P 4.5-3 The currents i_1 and i_2 in Figure P 4.5-3 are the mesh currents. Determine the value of the resistance R required to cause $v_a = -6$ V.
Answer: $R = 4\ \Omega$

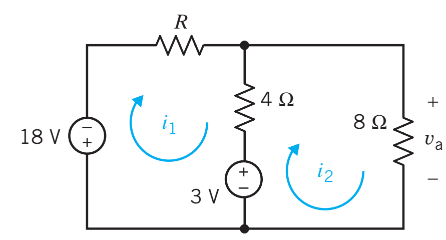

FIGURE P 4.5-3

P 4.5-4 Determine the mesh currents, i_a and i_b, in the circuit shown in Figure P 4.5-4.

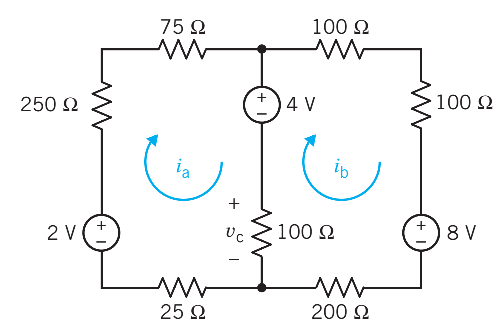

FIGURE P 4.5-4

P 4.5-5 Find the current i for the circuit of Figure P 4.5-5.
Hint: A short circuit can be treated as a 0-V voltage source.

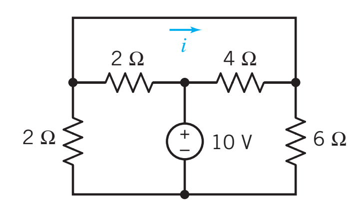

FIGURE P 4.5-5

P 4.5-6 Simplify the circuit shown in Figure P 4.5-6 by replacing series and parallel resistors by equivalent resistors. Next, analyze the simplified circuit by writing and solving mesh equations. (a) Determine the power supplied by each source. (b) Determine the power absorbed by the 30-Ω resistor.

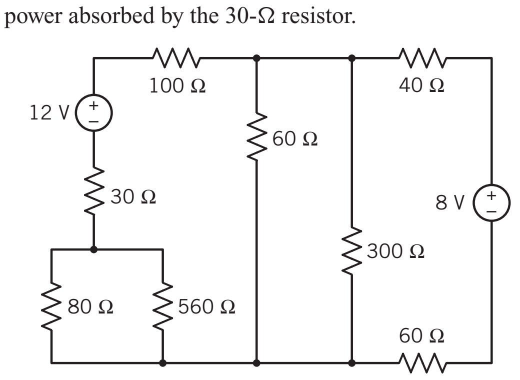

FIGURE P 4.5-6

P1: IML/OVY P2: IML/OVY QC: IML/OVY T1: IML
GTBL002-dorfc04 GTBL002-Dorf-v32 October 17, 2005 14:13

Section 4.6 Mesh Current Analysis with Current and Voltage Sources

P 4.6-1 Find i_b for the circuit shown in Figure P 4.6-1.
Answer: $i_b = 0.6$ A

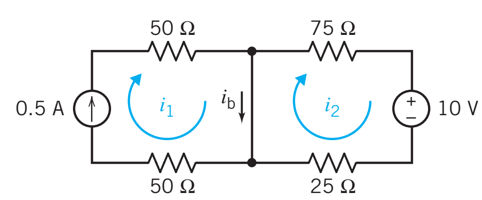

FIGURE P 4.6-1

P 4.6-2 Find v_c for the circuit shown in Figure P 4.6-2.
Answer: $v_c = 15$ V

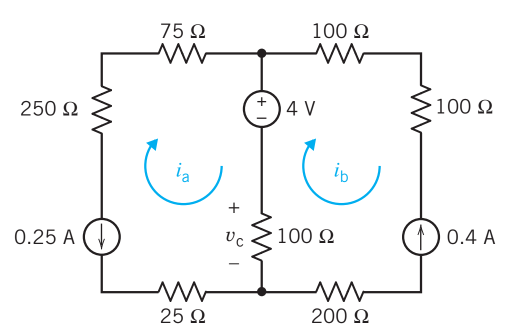

FIGURE P 4.6-2

P 4.6-3 Find v_2 for the circuit shown in Figure P 4.6-3.
Answer: $v_2 = 2$ V

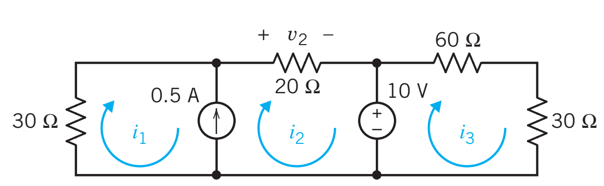

FIGURE P 4.6-3

P 4.6-4 Find v_c for the circuit shown in Figure P 4.6-4.

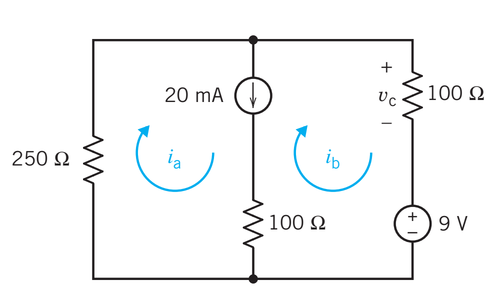

FIGURE P 4.6-4

P 4.6-5 Determine the value of the voltage measured by the voltmeter in Figure P 4.6-5.
Answer: 8 V

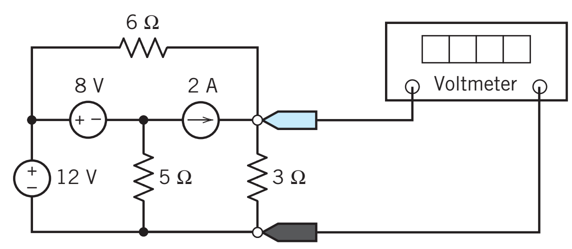

FIGURE P 4.6-5

P 4.6-6 Determine the value of the current measured by the ammeter in Figure P 4.6-6.
Hint: Write and solve a single mesh equation.
Answer: −5/6 A

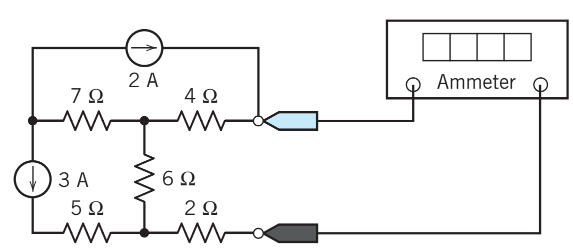

FIGURE P 4.6-6

P 4.6-7 The currents i_1, i_2, and i_3 in Figure P 4.6-7 are the mesh currents. Determine the value of the resistance R.

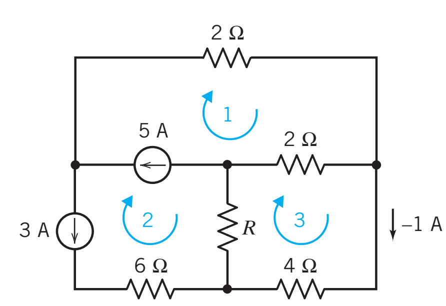

FIGURE P 4.6-7

P 4.6-8 Determine values of the mesh currents, i_1, i_2, and i_3, in the circuit shown in Figure P 6.7-8.

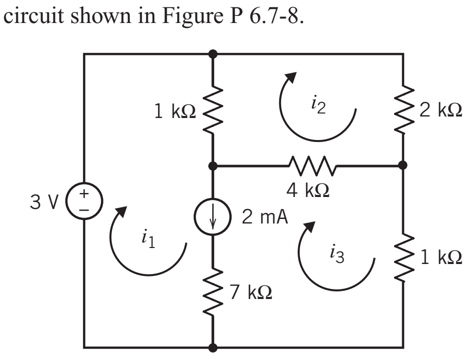

FIGURE P 4.6-8

***P 4.6-9** The circuit shown in Figure P 4.6-9 has three inputs: i_x, i_y and v_z. The output of the circuit is i_o. The output is related to the inputs by

$$i_o = a\, i_x + b\, i_y + c\, v_z$$

where a, b, and c are constants. Determine the values of a, b, and c.

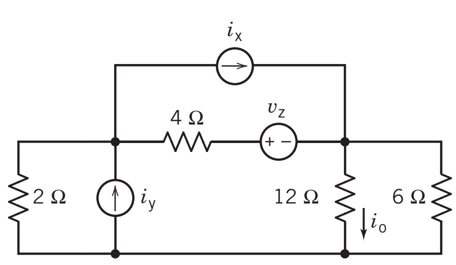

FIGURE P 4.6-9

P 4.6-10 The mesh currents in the circuit shown in Figure P 4.6-10 are

$$i_1 = -2.2213\text{ A},\ i_2 = 0.7787\text{ A, and } i_3 = 0.0770\text{ A}$$

(a) Determine the values of the resistances R_1 and R_3.
(b) Determine the value of the power supplied by the current source.

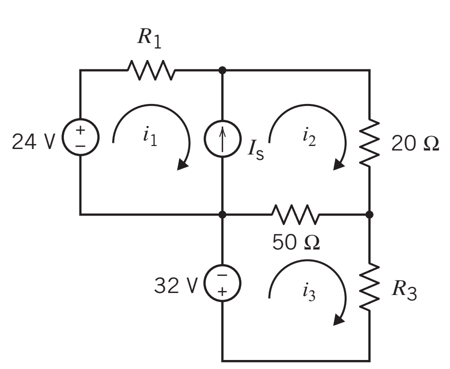

FIGURE P 4.6-10

P 4.6-11 Determine the value of the voltage measured by the voltmeter in Figure P 4.6-11.
Hint: Apply KVL to a supermesh to determine the current in the 2-Ω resistor.
Answer: 4/3 V

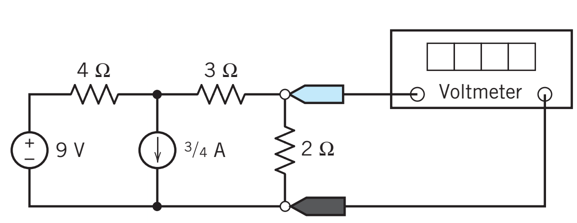

FIGURE P 4.6-11

P 4.6-12 Determine the value of the current measured by the ammeter in Figure P 4.6-12.
Hint: Apply KVL to a supermesh.
Answer: −0.333 A

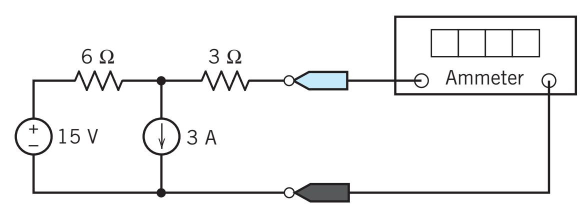

FIGURE P 4.6-12

Section 4.7 Mesh Current Analysis with Dependent Sources

P 4.7-1 Find v_2 for the circuit shown in Figure P 4.7-1.
Answer: $v_2 = 10$ V

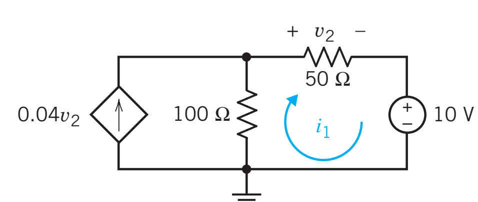

FIGURE P 4.7-1

P 4.7-2 Determine the mesh current i_a for the circuit shown in Figure P 4.7-2.
Answer: $i_a = -48$ mA

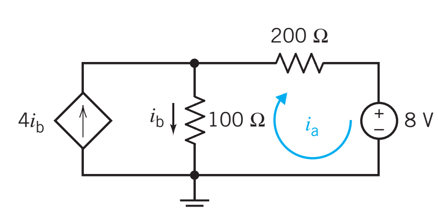

FIGURE P 4.7-2

P 4.7-3 Find v_o for the circuit shown in Figure P 4.7-3.
Answer: $v_o = 2.5$ V

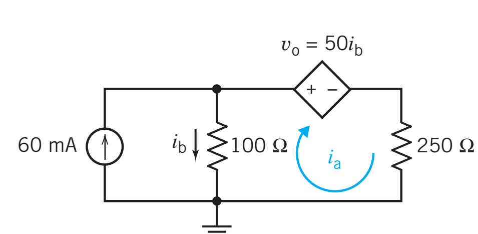

FIGURE P 4.7-3

P1: IML/OVY P2: IML/OVY QC: IML/OVY T1: IML
GTBL002-dorfc04 GTBL002-Dorf-v32 October 17, 2005 14:13

P 4.7-4 Determine the mesh current i_a for the circuit shown in Figure P 4.7-4.

Answer: $i_a = -24$ mA

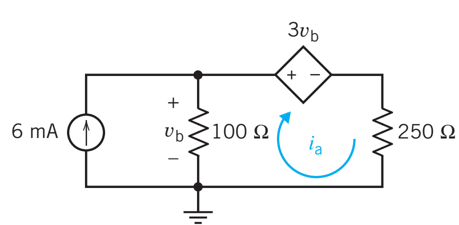

FIGURE P 4.7-4

P 4.7-5 Although scientists continue to debate exactly why and how it works, the process of utilizing electricity to aid in the repair and growth of bones—which has been used mainly with fractures—may soon be extended to an array of other problems, ranging from osteoporosis and osteoarthritis to spinal fusions and skin ulcers.

An electric current is applied to bone fractures that have not healed in the normal period of time. The process seeks to imitate natural electrical forces within the body. It takes only a small amount of electric stimulation to accelerate bone recovery. The direct current method uses an electrode that is implanted at the bone. This method has a success rate approaching 80 percent.

The implant is shown in Figure P 4.7-5*a* and the circuit model is shown in Figure P 4.7-5*b*. Find the energy delivered to the cathode during a 24-hour period. The cathode is represented by the dependent voltage source and the 100-kΩ resistor.

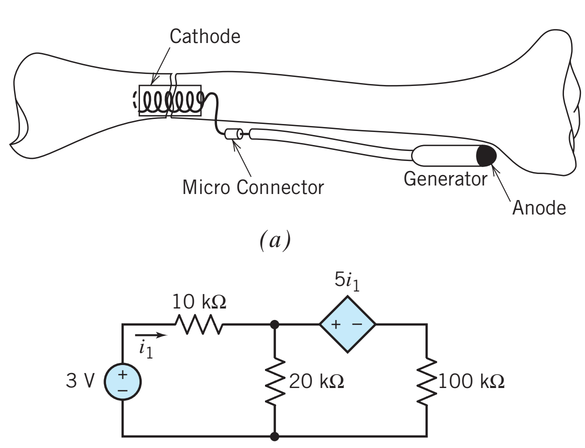

FIGURE P 4.7-5 (*a*) Electric aid to bone repair. (*b*) Circuit model.

P 4.7-6 The model of a bipolar junction transistor (BJT) amplifier is shown in Figure P 4.7-6.

(a) Determine the gain v_o/v_i.

(b) Calculate the required value of g in order to obtain a gain $v_o/v_i = -170$ when $R_L = 5$ kΩ, $R_1 = 100$ Ω, and $R_2 = 1$ kΩ.

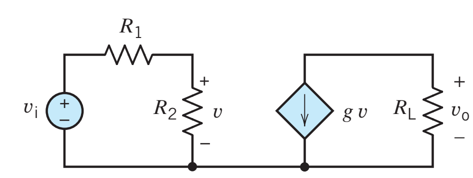

FIGURE P 4.7-6

P 4.7-7 The currents i_1, i_2 and i_3 are the mesh currents of the circuit shown in Figure P 4.7-7. Determine the values of i_1, i_2, and i_3.

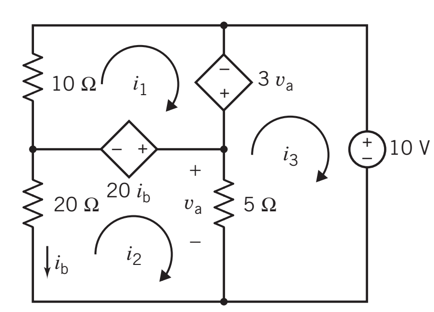

FIGURE P 4.7-7

P 4.7-8 Determine the value of the power supplied by the dependent source in Figure P 4.7-8.

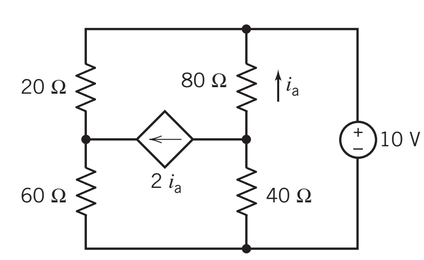

FIGURE P 4.7-8

P 4.7-9 Determine the value of the resistance R in the circuit shown in Figure P 4.7-9.

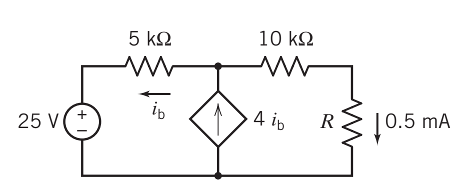

FIGURE P 4.7-9

P 4.7-10 The circuit shown in Figure P 4.7-10 is the small signal model of an amplifier. The input to the amplifier is the voltage source voltage, v_s. The output of the amplifier is the voltage v_o.

(a) The ratio of the output to the input, v_o/v_s, is called the gain of the amplifier. Determine the gain of the amplifier.

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(b) The ratio of the current of the input source to the input voltage, i_b/v_s, is called the input resistance of the amplifier. Determine the input resistance.

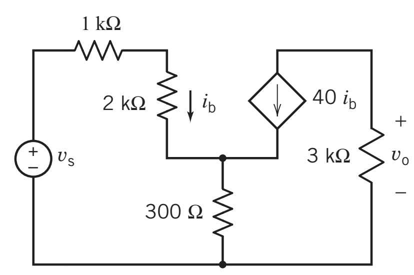

FIGURE P 4.7-10

P 4.7-11 Determine values of the mesh currents i_1, i_2, i_3, and i_4 in the circuit shown in Figure P 4.7-11.

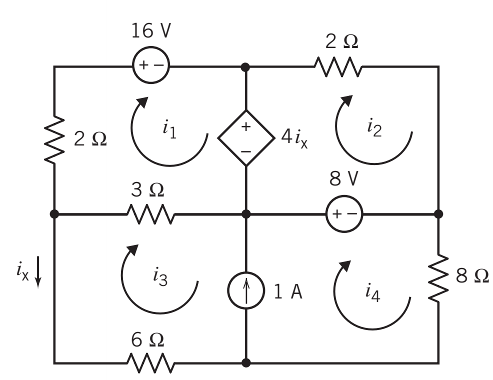

FIGURE P 4.7-11

P 4.7-12 Determine the values of the mesh currents of the circuit shown in Figure P 4.7-12.

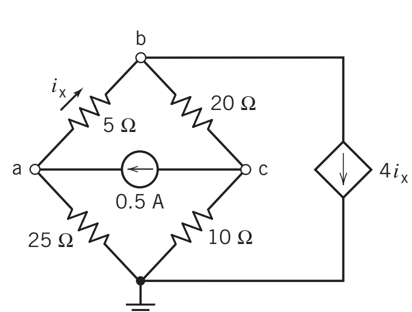

FIGURE P 4.7-12

P 4.7-13 The currents i_1, i_2, and i_3 are the mesh currents corresponding to meshes 1, 2, and 3 in Figure P 4.7-13. Determine the values of these mesh currents.

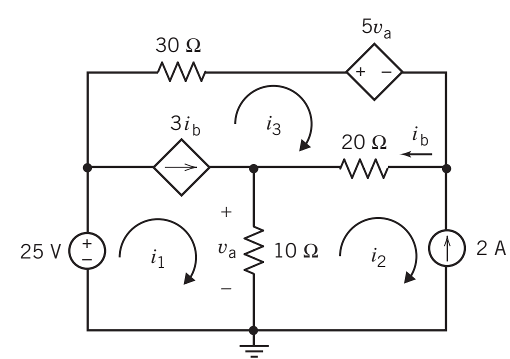

FIGURE P 4.7-13

P 4.7-14 The currents i_1, i_2, and i_3 are the mesh currents corresponding to meshes 1, 2, and 3 in Figure P 4.7-14. The values of these currents are

$$i_1 = -1.375\text{ A}, i_2 = -2.5\text{ A}, \quad \text{and} \quad i_3 = -3.25\text{ A}$$

Determine the values of the gains of the dependent source, A and B.

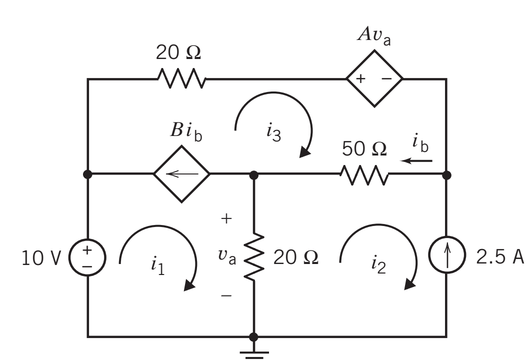

FIGURE P 4.7-14

P 4.7-15 Determine the current i in the circuit shown in Figure P 4.7-15.
Answer: $i = 3$ A

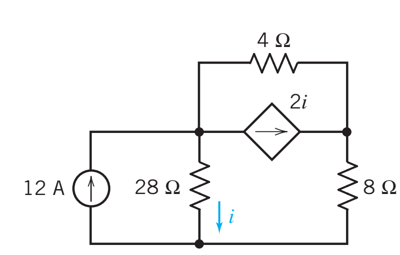

FIGURE P 4.7-15

Section 4.8 The Node Voltage Method and Mesh Current Method Compared

***P 4.8-1** The circuit shown in Figure P 4.8-1 has two inputs, the voltage source voltages, v_1 and v_2. The circuit has one output, the dependent source voltage, v_o. Design this circuit so that the output is related to the inputs by

$$v_o = 2v_1 + 0.5v_2$$

Hint: Determine the required values of A, R_1, R_2, R_3, and R_4.

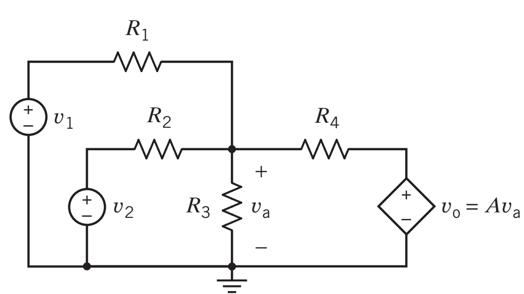

FIGURE P 4.8-1

P 4.8-2 The circuit shown in Figure P 4.8-2 has two inputs, v_s and i_s, and one output v_o. The output is related to the inputs by the equation

$$v_o = ai_s + bv_s$$

where a and b are constants to be determined. Determine the values a and b by (a) writing and solving mesh equations and (b) writing and solving node equations.

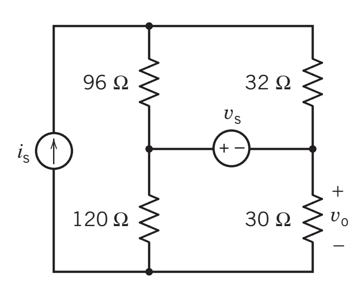

FIGURE P 4.8-2

P 4.8-3 Determine the power supplied by the dependent source in the circuit shown in Figure P 4.8-3 by writing and solving (a) node equations and (b) mesh equations.

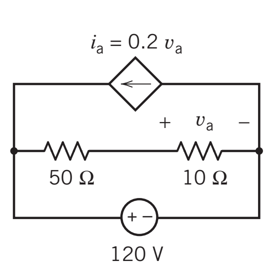

FIGURE P 4.8-3

Section 4.10 How Can We Check . . . ?

P 4.10-1 Computer analysis of the circuit shown in Figure P 4.10-1 indicates that the node voltages are $v_a = 5.2$ V, $v_b = -4.8$ V, and $v_c = 3.0$ V. Is this analysis correct?
Hint: Use the node voltages to calculate all the element currents. Check to see that KCL is satisfied at each node.

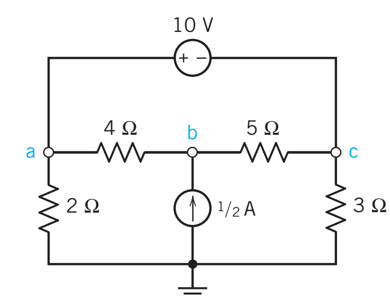

FIGURE P 4.10-1

P 4.10-2 An old lab report asserts that the node voltages of the circuit of Figure P 4.10-2 are $v_a = 4$ V, $v_b = 20$ V, and $v_c = 12$ V. Are these correct?

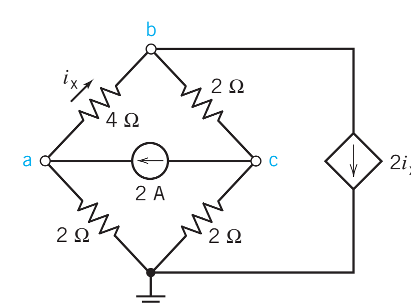

FIGURE P 4.10-2

P 4.10-3 Your lab partner forgot to record the values of R_1, R_2, and R_3. He thinks that two of the resistors in Figure P 4.10-3 had values of 10 kΩ and that the other had a value of 5 kΩ. Is this possible? Which resistor is the 5-kΩ resistor?

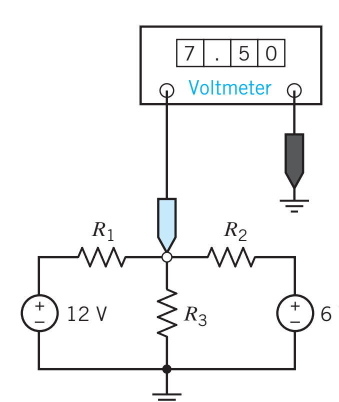

FIGURE P 4.10-3

P 4.10-4 Computer analysis of the circuit shown in Figure P 4.10-4 indicates that the node voltages are $v_1 = -8$ V, $v_2 = -20$ V, and $v_3 = -6$ V. Verify that this analysis is correct.
Hint: Use the node voltages to calculate the element currents. Verify that KCL is satisfied at each node.

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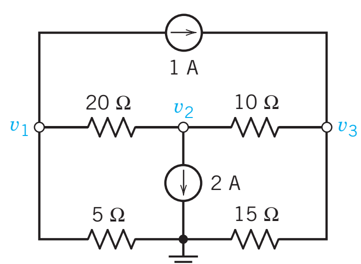

FIGURE P 4.10-4

P 4.10-5 Computer analysis of the circuit shown in Figure P 4.10-5 indicates that the mesh currents are $i_1 = 2$ A, $i_2 = 4$ A, and $i_3 = 3$ A. Verify that this analysis is correct.

Hint: Use the mesh currents to calculate the element voltages. Verify that KVL is satisfied for each mesh.

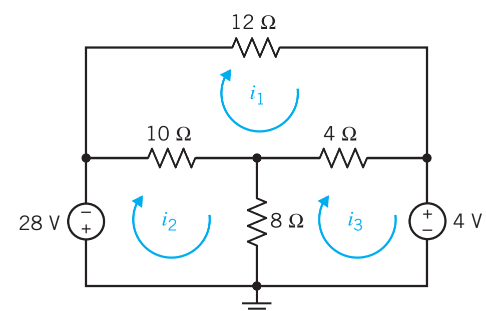

FIGURE P 4.10-5

PSpice Problems

SP 4-1 Use PSpice to determine the node voltages of the circuit shown in Figure SP 4-1.

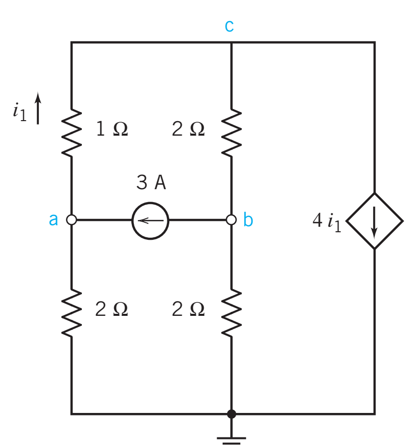

FIGURE SP 4-1

SP 4-2 Use PSpice to determine the mesh currents of the circuit shown in Figure SP 4-2.

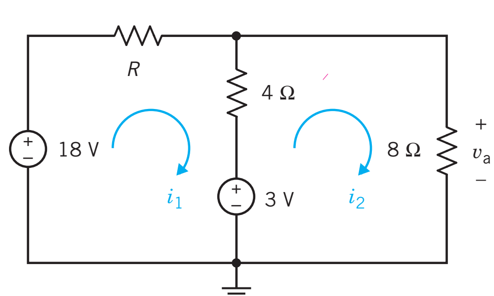

FIGURE SP 4-2

SP 4-3 The voltages v_a, v_b, v_c, and v_d in Figure SP 4-3 are the node voltages corresponding to nodes a, b, c and d. The current i is the current in a short circuit connected between nodes b and c. Use PSpice to determine the values of v_a, v_b, v_c, and v_d and of i.

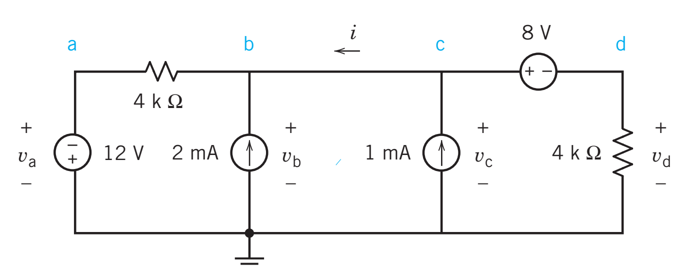

FIGURE SP 4-3

SP 4-4 Determine the current, i, shown in Figure SP 4-4.
Answer: $i = 0.56$ A

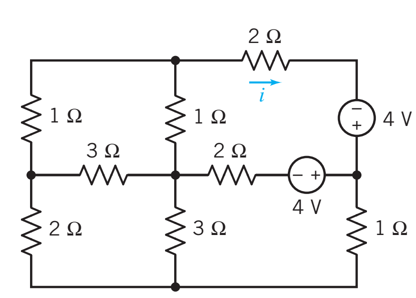

FIGURE SP 4-4

Design Problems

DP 4-1 An electronic instrument incorporates a 15-V power supply. A digital display is added that requires a 5-V power supply. Unfortunately, the project is over budget and you are instructed to use the existing power supply. Using a voltage divider, as shown in Figure DP 4-1, you are able to obtain 5 V. The specification sheet for the digital display shows that the display will operate properly over a supply voltage range of 4.8 V to 5.4 V. Furthermore, the display will draw 300 mA (I) when the display is active and 100 mA when quiescent (no activity).

(a) Select values of R_1 and R_2 so that the display will be supplied with 4.8 V to 5.4 V under all conditions of current I.

(b) Calculate the maximum power dissipated by each resistor, R_1 and R_2, and the maximum current drawn from the 15-V supply.

(c) Is the use of the voltage divider a good engineering solution? If not, why? What problems might arise?

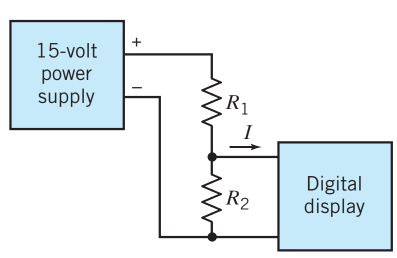

FIGURE DP 4-1

DP 4-2 For the circuit shown in Figure DP 4-2, it is desired to set the voltage at node a equal to 0 V in order to control an electric motor. Select voltages v_1 and v_2 in order to achieve $v_a = 0$ V when v_1 and v_2 are less than 20 V and greater than zero and $R = 2\ \Omega$.

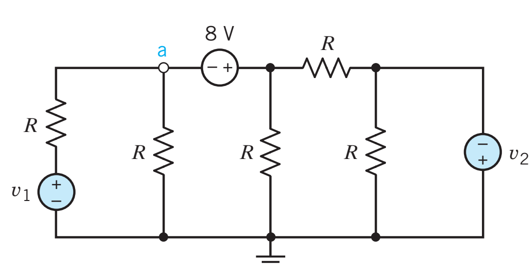

FIGURE DP 4-2

DP 4-3 A wiring circuit for a special lamp in a home is shown in Figure DP 4-3. The lamp has a resistance of 2 Ω, and the designer selects $R = 100\ \Omega$. The lamp will light when $I \geq 50$ mA but will burn out when $I > 75$ mA.

(a) Determine the current in the lamp and determine if it will light for $R = 100\ \Omega$.

(b) Select R so that the lamp will light but will not burn out if R changes by ±10 percent because of temperature changes in the home.

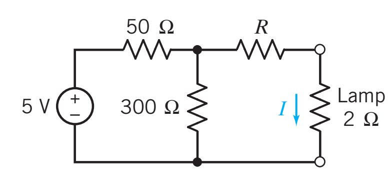

FIGURE DP 4-3 A lamp circuit.

DP 4-4 In order to control a device using the circuit shown in Figure DP 4-4, it is necessary that $v_{ab} = 10$ V. Select the resistors when it is required that all resistors be greater than 1 Ω and $R_3 + R_4 = 20\ \Omega$.

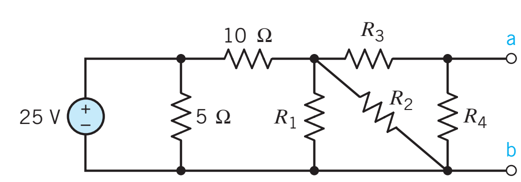

FIGURE DP 4-4

DP 4-5 The current i shown in the circuit of Figure DP 4-5 is used to measure the stress between two sides of an earth fault line. Voltage v_1 is obtained from one side of the fault, and v_2 is obtained from the other side of the fault. Select the resistances R_1, R_2, and R_3 so that the magnitude of the current i will remain in the range between 0.5 mA and 2 mA when v_1 and v_2 may each vary independently between +1 V and +2 V ($1\text{ V} \leq v_n \leq 2\text{ V}$).

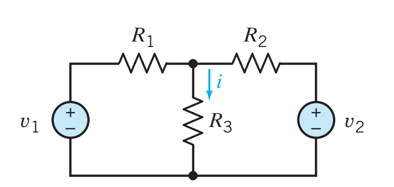

FIGURE DP 4-5 A circuit for earth fault line stress measurement.